REGULAR SCHOOL BOARD MEETING

November 15, 2022

7:00 PM

Educational Support Center
Board Meeting Room
3600-52nd Street
Kenosha, Wisconsin

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Kenosha Unified School District

Regular School Board Meeting
November 15, 2022
Educational Support Center
3600 52nd St.
Kenosha, WI 53144
7:00 PM

Kenosha Unified School District

Kenosha, WI

November 15, 2022

The Office of Human Resources recommends the following actions:

ACTION	LAST NAME	FIRST NAME	SCHOOL/DEPT	POSITION	STAFF	DATE	FTE
Appointment	Andersen	Jennifer	Vernon Elementary School	Grade 5	Instructional	11/01/2022	1
Appointment	Baldwin	Jourdan	Bradford High School	SEL Intervention Specialist	Instructional	11/07/2022	1
Appointment	Barrett	Abigail	Indian Trail HS & Academy	Cross Categorical	Instructional	10/24/2022	1
Appointment	Barton	Terra	Lincoln Middle School	Administrative Support (10 MO)	ASP	11/07/2022	1
Appointment	Cetnarowski	Cheri	Washington Middle School	Intervention Specialist	Instructional	11/01/2022	1
Appointment	Cooper II	Charles	Indian Trail HS & Academy	Assistant Head Custodian	Facilities	11/04/2022	1
Appointment	Dean	Cortney	Lincoln Middle School	Grade 6	Instructional	11/14/2022	1
Appointment	Donovan	Benjamin	Indian Trail HS & Academy	Security	ESP	11/08/2022	1
Appointment	Dorey	Marie	Curtis Strange Elementary School	Math Intervention Specialist	Instructional	11/21/2022	1
Appointment	Eckel	Jennifer	Bradford High School	Cross Categorical	Instructional	11/07/2022	1
Appointment	Eisenhauer	Kelly	Stocker Elementary School	Kindergarten	Instructional	11/21/2022	1
Appointment	Green	Angel	EBSOLA CA	4K Program	Instructional	11/07/2022	1
Appointment	Kelly	Thomas	Jefferson Elementary School	Grade 3	Instructional	10/28/2022	1
Appointment	Koeller	Eric	Bradford High School	Math	Instructional	11/07/2022	1
Appointment	Mahlandt	Patti	Stocker Elementary School	Special Education	ESP	10/17/2022	1
Appointment	Reed	Desiree	Indian Trail HS & Academy	Special Education	ESP	11/07/2022	1
Appointment	Tallman	Gabriella	Lance Middle School	Special Education	ESP	10/24/2022	1
Appointment	Wember	Ted	Vernon/Southport Elementary Schools	Night Custodian Second Shift	Facilities	11/07/2022	1
Appointment	Widmar	Kristy	Grewenow Elementary School	Head Start	ESP	10/17/2022	1
Resignation	Bates	Justine	Frank Elementary School	E.C.	Instructional	11/18/2022	1
Resignation	Brockman	Nathaniel	Office of Student Support	Speech Therapist	Instructional	10/28/2022	1
Resignation	Gates	Bethany	McKinley Elementary School	Kindergarten	Instructional	10/28/2022	1
Resignation	Jernigan	Darnell	Forest Park Elementary School	Head Custodian	Facilities	10/21/2022	1
Resignation	June	Rachel	Lance Middle School	Special Education	ESP	10/25/2022	1
Resignation	Kinney	Sarah	Bullen Middle School	Special Education	ESP	10/18/2022	1
Resignation	Maravilla	Michelle	Indian Trail HS & Academy	Security	ESP	10/14/2022	1
Resignation	Quinn	Jessica	Head Start	Family Service Provider	ESP	11/10/2022	1
Resignation	Schneider	Christa	Indian Trail HS & Academy	Administrative Specialist (12 MO)	ASP	11/11/2022	1
Resignation	Zielsdorf	Vicki	Forest Park Elementary School	Administrative Specialist (10 MO)	ASP	10/28/2022	1
Retirement	Mastronardi	Steven	Facilities	Custodial Supervisor	Facilities	12/02/2022	1
Retirement	Zastrow	Tracy	Curtis Strange Elementary School	Speech Therapist	Instructional	12/31/2022	1
Separation	Brooks	Julia	Washington Middle School	MS Choir/Performing Arts	Instructional	10/14/2022	1

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SPECIAL MEETING & EXECUTIVE SESSION OF THE KENOSHA UNIFIED SCHOOL BOARD HELD OCTOBER 11, 2022

A special meeting of the Kenosha Unified School Board was held on Tuesday, October 11, 2022, in Room 125 at the Educational Support Center. The purpose of this meeting was to vote on holding an executive session to follow immediately.

The meeting was called to order at 6:45 P.M. with the following members present: Mrs. Schmaling, Mr. Price, Mr. Meadows, Ms. Stevens, Mrs. Modder, and Ms. Adams. Dr. Weiss, Attorney Dawson (via telephone), Mr. Hamdan, Mrs. Ruder, and Mr. Keckler were also present. Mr. Battle was excused.

Ms. Adams, President, opened the meeting by announcing that this was a special meeting of the School Board of the Kenosha Unified School District. Notice of this special meeting was given to the public by forwarding a copy of the notice to all requesting radio stations and newspapers.

Ms. Adams announced that an executive session had been scheduled to follow this special meeting for the purpose of Personnel: Compensation and/or Contracts and Litigation.

Mrs. Modder moved that the executive session be held. Ms. Stevens seconded the motion.

Roll call vote. Ayes: Mrs. Schmaling, Mr. Price, Mr. Meadows, Ms. Stevens, Mrs. Modder, and Ms. Adams. Noes: None. Unanimously approved.

1. <u>Litigation</u>

Attorney Blair Dawson from McDonald Hopkins LLC, updated board members on a potential legal matter.

Attorney Dawson; Mr. Tarik Hamdan, Chief Financial Officer; Mrs. Tanya Ruder, Chief Communications Officer; and Mr. Kristopher Keckler, Chief Information Officer, answered questions from board member.

Attorney Dawson departed the meeting at 7:16 P.M.

Mrs. Ruder and Mr. Keckler departed the meeting at 7:27 P.M.

2. <u>Personnel: Compensation and/or Contract</u>

Dr. Weiss and board members discussed an item related to the superintendent's contact.

Meeting adjourned at 7:29 P.M.

Stacy Stephens
School Board Secretary

SPECIAL MEETING & EXECUTIVE SESSION
OF THE KENOSHA UNIFIED SCHOOL BOARD
HELD OCTOBER 25, 2022

A special meeting of the Kenosha Unified School Board was held on Tuesday, October 25, 2022, in Room 125 at the Educational Support Center. The purpose of this meeting was to vote on holding an executive session to follow immediately.

The meeting was called to order at 6:03 P.M. with the following members present: Mrs. Schmaling, Mr. Price, Mr. Battle, Ms. Stevens (virtual), Mrs. Modder, and Ms. Adams. Dr. Weiss and Mr. Neir were also present. Mr. Meadows was excused.

Ms. Adams, President, opened the meeting by announcing that this was a special meeting of the School Board of the Kenosha Unified School District. Notice of this special meeting was given to the public by forwarding a copy of the notice to all requesting radio stations and newspapers.

Ms. Adams announced that an executive session had been scheduled to follow this special meeting for the purpose of Review of Findings/Order by Independent Hearing Officer, Litigation, Personnel: Problems, and Personnel: Evaluation Consideration.

Mrs. Modder moved that the executive session be held. Mr. Price seconded the motion.

Roll call vote. Ayes: Mrs. Schmaling, Mr. Price, Mr. Battle, Ms. Stevens, Mrs. Modder, and Ms. Adams. Noes: None. Unanimously approved.

1. Review of Findings/Order by Independent Hearing Officer

Mr. Anthony Casper, Administrative Review Committee Chairperson, arrived at 6:04 P.M. and presented board members with information pertaining to a student expulsion.

Dr. Weiss, Mr. Neir, and Mr. Casper departed the meeting at 6:14 P.M.

Mrs. Modder moved to approve the hearing officer's recommendation in regards to the student expulsion. Mr. Battle seconded the motion. Unanimously approved.

Dr. Weiss and Mr. Neir returned to the meeting at 6:16 P.M.

2. Personnel: Problems

Mr. Neir updated board members on a few employee relations cases.

Mr. Neir was excused from the meeting at 6:20 P.M.

3. Litigation

Mr. Kristopher Keckler, Chief Information Officer, and Mrs. Tanya Ruder, Chief Communications Officer, arrived at 6:21 P.M. and updated board members on a potential legal matter.

Mr. Keckler and Mrs. Ruder were excused from the meeting at 6:31 P.M.

4. Personnel: Evaluation Consideration

Dr. Weiss and board members discussed the evaluation tool to be utilized for the superintendent's evaluation.

Mr. Price moved to adjourn the meeting. Mrs. Modder seconded the motion. Unanimously approved.

Meeting adjourned at 6:40 P.M.

Stacy Stephens
School Board Secretary

REGULAR MEETING OF
THE KENOSHA UNIFIED SCHOOL BOARD
HELD OCTOBER 25, 2022

A regular meeting of the Kenosha Unified School Board was held on Tuesday, October 25, 2022, at 7:00 P.M. in the Board Room of the Educational Support Center. Ms. Adams, President, presided.

The meeting was called to order at 7:04 P.M. with the following Board members present: Mrs. Schmaling, Mr. Price, Mr. Battle, Ms. Stevens (virtual), Mrs. Modder, and Ms. Adams. Dr. Weiss was also present. Mr. Meadows was excused.

Ms. Adams, President, opened the meeting by announcing that this was a regular meeting of the School Board of Kenosha Unified School District. Notice of this regular meeting was given to the public by forwarding the complete agenda to all requesting radio stations and newspapers. Copies of the complete agenda are available for inspection at all public schools and the Superintendent's office. Anyone desiring information as to forthcoming meetings should contact the Superintendent's office.

Mrs. Schmaling introduced the student ambassador, Zulikha Gondal from Bradford High School, and she made her comments.

There were no Administrative and Supervisory Appointments.

Dr. Weiss gave the Superintendent's report.

Mr. Price gave the legislative report.

Views and comments were made by the public.

Remarks by the President were made by Ms. Adams.

Board members considered the following Consent-Approve items:

Consent-Approve item X-A – Recommendations Concerning Appointments, Leaves of Absence, Retirements, Resignations, and Separations.

Consent-Approve item X-B – Minutes of the 9/20/22 Public Hearing on the Budget, 9/20/22 Annual Meeting of Electors, 9/20/22 Special Meeting, 9/27/22 Special Meeting and Executive Session, and 9/27/22 Regular Meeting.

Consent-Approve item X-C – Summary of Receipts, Wire Transfers, and Check Registers submitted by Mrs. Lisa Salo, Accounting Manager; Mr. Tarik Hamdan, Chief Financial Officer; and Dr. Ormseth, excerpts follow:

"It is recommended that the September 2022 cash receipt deposits totaling $200,949.08, and cash receipt wire transfers-in totaling $24,677,663.29, be approved.

Check numbers 615721 through 616728 (net of voided batches) totaling $6,482,597.57, and general operating wire transfers-out totaling $4,172,832.17, are recommended for approval as the payments made are within budgeted allocations for the respective programs and projects.

It is recommended that the September 2022 net payroll and benefit EFT batches totaling $12,459,278.47, and net payroll check batches totaling $11,019.30, be approved."

Consent-Approve item X-D – Proposed Changes to KUSD Policy 5210 – Entrance Age submitted by Ms. Luanne Rohde, Director of Early Education; Mr. William Haithcock, Chief of School Leadership; Mr. Kristopher Keckler, Chief Information Officer; and Dr. Weiss, excerpts follow:

"Kenosha Unified Policy 5210 establishes the entrance age for all students who wish to enroll within the district. Based on current state of Wisconsin statutes, the marker date of September 1 is used to identify the age of the student in relation to the expected grade level placement. This policy was last updated in the spring of 2011, and with recent impact from the COVID pandemic, as well as clarification on the historical district practices, some updated language has been provided to help clarify this process for staff and parents.

The last few years has shown a large fluctuation in early education enrollment practices for nearly every school district. Many parents kept students out of public education during the early years of the COVID pandemic, and grades 4K and K are not required attendance ages/grade levels. Due to this increased evidence of delayed enrollment practices, the Wisconsin Department of Public Instruction has worked to promote equalized enrollment based mainly from the age of the student for early education classes.

At its September 27, 2022, meeting the Board of Education approved the proposed changes to KUSD Policy 5210 Entrance Age as a first reading. Administration recommends that the School Board accept these as a second reading at its October 25, 2022, regular School Board meeting."

Mrs. Modder moved to approve the consent agenda as presented. Mr. Price seconded the motion. Unanimously approved.

Mr. Finnemore, Director of Facilities, presented the School Safety Law Requirements submitted by himself, Mr. Haithcock, and Dr. Weiss, excerpts follow:

"On March 26, 2018, the former Governor signed the Wisconsin School Safety Bill which created a new Office of School Safety in the Department of Justice (DOJ) and provided $100M in funding for school safety grants of which KUSD received $2,121,287. In addition to the grant funding, the new school safety law contained other requirements some of which require School Board review and approval. This report will serve as the formal Board approval of these requirements for 2022.

The law requires (State Statute 118.07 (4) (b, c, d & e)) that school districts file a copy of their school safety plan with the Department of Justice Office of School Safety prior to January 1, 2019, and before January 1 every year thereafter. We will submit our safety plan as part of the annual submittals to the DOJ at the end of December. The law also requires that the School Board review and approve the plan every three years, and that local law enforcement also review the plan. Our last formal review and approval was done in 2021, therefore a formal review and approval is not necessarily required this year; however, we did make one change of note as well as the annual updating of contact names. The change of note was the creation of a new procedure for a HOLD which was something we piloted last year and determined that we should make it part of our District procedures.

Administration recommends Board approval of the School Safety Law Requirements as described in this report."

Mr. Battle moved to approve the School Safety Law Requirements as described in this report. Mrs. Schmaling seconded the motion. Unanimously approved.

Mr. Finnemore presented the Bong Property Sale submitted himself, Mr. Hamdan, and Dr. Weiss, excerpts follow:

"In December of 2019, Administration provided the Board with a summary of KUSD owned vacant properties and whether those properties were viewed as future school sites or sites that should be sold or traded for other properties. We recommended, and the Board concurred, that the district pursue the sale of the approximately 132 acres of land adjacent and across the street from Richard Bong State Park (Attachment 1). We had an appraisal performed on the property by Pitts Brothers of Kenosha in October of 2020, and that appraisal valued the land at $1,300,000. After the appraisal was complete, we discussed in an Executive Session the concept of selling the property to the State of Wisconsin DNR and/or non-profit organizations affiliated with the DNR to preserve the land and allow for continued recreational use by the public.

KUSD began discussions with the DNR and a non-profit group that works with the DNR at Bong, the Bong Naturalist Association (BNA), in February of 2021. The BNA presented a financial plan to secure private and public funding over the course of an approximately 16-month process. Administration provided the Board another update in an Executive Session on July 27, 2021, summarizing the BNA financial plan and an update on where they were at in regards to securing the necessary funding. The Board made no commitments at that time, but continued to express an interest in working with the BNA as well as for selling the land near Bong State Park.

BNA/DU made an initial verbal offer to purchase the property in late June of this year. That offer was brought forward to the Board in Executive Session on June 28, 2022. After two weeks of negotiations, the KUSD Board and Administration negotiated a sale price of $1,200,000 with BNA/DU. This price was approximately the average of the three appraisals.

Administration recommends Board approval of the sale of the KUSD owned property adjacent to and across the street from Richard Bong State Park to Wetlands America Trust, Inc. (a supporting organization of Ducks Unlimited, Inc. and with recognition of their partnership with the Bong Naturalist Association) for $1,200,000 as described in this report."

Mr. Battle moved to approve the sale of the KUSD owned property adjacent to and across the street from Richard Bong State Park to Wetlands America Trust, Inc. (a supporting organization of Ducks Unlimited, Inc. and with recognition of their partnership with the Bong Naturalist Association) for $1,200,000 as described in this report. Ms. Stevens seconded the motion. Unanimously approved.

Mr. Hamdan presented the Closing of Pleasant Prairie Tax Incremental District (TID) #2 submitted by himself, Mr. Finnemore, and Dr. Weiss, excerpts follow:

"Tax Incremental Financing (TIF) is a financing option that allows a municipality (town, village, or city) to fund infrastructure and other improvements, through property tax revenue on newly developed property. The municipality identifies an area, which is designated as the Tax Incremental District (TID), as appropriate for a certain type of development. The municipality identifies projects to encourage and facilitate the desired development. Then as property values rise, the municipality uses the property tax paid on that development to pay for the projects. After the project costs are paid, the municipality closes the TID. The municipality, schools, county, and technical college are able to levy taxes on the value of the new development for the areas that fall within their boundaries.

TIF use varies depending on the project and the municipality. In some cases, the municipality chooses an area it would like to develop or that is unlikely to develop without assistance. Then the municipality designs improvements (ex: roads, sidewalks, sewer systems) to attract growth. In other cases, a developer or company identifies a site where they might locate. A developer may also negotiate with the municipality to use TIF to fund some improvements (ex: demolition, soil clean up, roads, water) the developer needs. Either way, an area facing development challenges receives help to grow. This creates a larger tax base for the municipality and the overlying taxing jurisdictions such as schools, counties, and technical colleges (WI Department of Revenue – Tax Incremental Financing Manual, pg. 3)

State law requires a Joint Review Board (JRB) to oversee Tax Incremental Districts (TIDs). State law defines its members and responsibilities. JRB members represent a taxing jurisdiction (municipality, county, school, technical college). It is the JRB's responsibility to approve or deny the creation or amendment of a TID. When the JRB approves a TID, it agrees on the development needs for Tax Incremental Financing (TIF) to proceed. The JRB jurisdictions agree to sacrifice some amount of tax revenue for years into the future expecting the tax base will ultimately increase (WI Department of Revenue – Tax Incremental Financing Manual, pg. 26). The Kenosha Unified School District is represented by Ms. Yolanda Adams, School Board President.

The municipality must terminate the TID at the end of the maximum life, or when the tax increments collected exceed the approved project costs. At termination, the entire TID's property value is available for the overlying taxing jurisdictions to tax. Aside from property value change, if the tax increment revenue exceeds the project costs at termination, the municipality must return the surplus revenue to the overlying taxing jurisdictions in proportion to the overlying taxing jurisdictions' respective tax levy without TIF (WI Department of Revenue – Tax Incremental Financing Manual, pg. 5).

The Pleasant Prairie TID #2 has officially closed with a surplus of $10,661,942. Following the Village TID Annual Meeting held on June 22, 2022, the expected distribution to KUSD is $3,540,822.

While the exact payment date is still unknown, we are expecting to receive the funds sometime before the end of the calendar year ending December 31, 2022. These funds will not be restricted so that the School Board has full authority to allocate them as they see fit.

Attachment A is a list of recommended projects identified by the administration as a good fit for this type of one-time funding should the Board decide to move in that direction.

This report is being provided as an informational update for discussion."

Mrs. Julie Housaman, Chief Academic Officer, presented the Request to Partner with the Wisconsin Department of Public Instruction on the Project AWARE Grant submitted by herself; Mrs. Jenny Schmidt, Director of Special Education & Student Support; and Dr. Weiss, excerpts follow:

"The Wisconsin Department of Public Instruction (DPI) is seeking to partner with Kenosha Unified School District in their application for the Project AWARE (Advancing Wellness and Resiliency in Education) Grant. Recipients of this funding will leverage their partnerships to implement mental health related promotion, awareness, prevention, intervention, and resilience activities to ensure that students have access and are connected to appropriate and effective behavioral health services.

The purpose of the Project AWARE grant is to develop a sustainable infrastructure for schoolbased mental health programs and services. This grant will enable a stronger partnership between the Kenosha Unified School District (KUSD), the Department of Public Instruction (DPI) and local community agency partners. KUSD would like to continue to expand our continuum of services for students and families with the resources available through the Project AWARE grant. The project AWARE grant project will be implemented over a span of four years. The population of focus is school-aged youth (i.e., children and youth in grades K-12).

A requirement of the AWARE grant includes the creation and implementation of a school-based mental health intervention system that is based on a three-tiered public health model: (Tier 1) universal prevention and mental health promotion; (Tier 2) secondary prevention and brief intervention services; and (Tier 3) tertiary intervention and behavioral health treatment. The three-tiered approach must be culturally competent, grief and trauma-informed, developmentally appropriate, evidence-based, or evidence-informed, and address the mental health effects of COVID-19.

According to the National Alliance on Mental Illness, 1 in 5 children experience a mental health condition; 20 percent of children ages 14 and up with a mental health condition will drop out of 56 school. Suicide is the second leading cause of death for 15 to 24 year olds. This has become an ever-increasing challenge for students, families, and schools, presenting an additional barrier for many students to learning and future success.

In 2019-20 the Youth Risk Behavior Survey (YRBS) was given to high school students in Kenosha Unified School District at Bradford, Indian Trail, Reuther, and Tremper. Many of the student responses highlight the increasing concerns of students and the need for preventative and intervening services among the students in the Kenosha Unified School district.

Because of this, addressing barriers to learning, including mental health challenges, through comprehensive learning, social, emotional, and mental health supports, has become an essential function of schools. College-and-career readiness requires that graduates are not only academically prepared but also socially and emotionally competent. Therefore, addressing these challenges and the barriers to learning cannot be optional as social-emotional development, mental health, and academic achievement have a critical connection in the success of all students.

Total amount of grant funds requested: $2,000,000 or $500,000 per year for a span of 4 years.

Administration recommends that the school board approve administration's request to partner with DPI in submitting a proposal for the Project AWARE Grant and to implement the grant if received by the Department of Health and Human Services in collaboration with Wisconsin Department of Public Instruction."

Mrs. Modder moved to approve administration's request to partner with DPI in submitting a proposal for the Project AWARE Grant and to implement the grant if received by the Department of Health and Human Services in collaboration with Wisconsin Department of Public Instruction. Mr. Price seconded the motion. Unanimously approved.

Mr. Keckler presented the Official Third Friday Enrollment Report submitted by himself, Mrs. Laura Sawyer, Research Analyst; Mrs. Erin Roethe, Data Analyst; and Dr. Weiss, excerpts follow:

"Annually, Administration provides the Kenosha Unified School Board with the District's Official Third Friday Enrollment Report. The data contained in this report are also reported to the Wisconsin Department of Public Instruction (DPI) in its designated format. The School Board should note that this report contains only enrollment data and does not contain student membership data that are used to develop revenue projections and budgetary planning documents.

District-wide, enrollment decreased -433 students, from 19,620 students in 2021-22 to 19,187 in 2022-23. Beginning in 2009-10, Kenosha started to experience a decline in community birth rates, with the related effect of declines in elementary school enrollments five years later. This trend currently impacts grades pre-kindergarten through grade 7.

KUSD continues to expand its diverse student population. The combined nonwhite race/ethnicities make up a majority of the student population at 54.4%. The enrollment distribution for Asian, Black, American Indian, and Native Hawaiian/Pacific Islander remains constant. A continual increase can be seen in the number of students identified as Hispanic students.

The 2022-23 Official Third Friday Enrollment Report is an informational item."

Mr. Keckler gave a PowerPoint presentation entitled Official Student Enrollment Trend which covered the following topics: overall enrollment trends, projected U.S. births, KUSD boundary annual births, number of births and KUSD kindergarten enrollment, KUSD exits to local schools, local non-KUSD school impact, KUSD homeschool impact, KUSD open enrollment, KUSD race/ethnicity, K-5: 8 year trend, KUSD boundary elementary school enrollments, grades 6-8: 8 year trend, KUSD middle school enrollment, grades 9-12: 8 year trend, KUSD boundary high school enrollments, KUSD cohort average: 8 year trend, change in enrollments and teacher FTE, % change in enrollments and teacher FTE, 5 year % change in enrollments and teacher FTE, and KUSD class size average.

Mr. Hamdan presented the Change in the Fiscal 2021-22 Adopted Budget submitted by himself and Dr. Weiss, excerpts follow:

"The Board of Education adopted the 2021-2022 budget on October 26, 2021, as prescribed by Wisconsin State Statute 65.90. From time to time there is a need to modify or amend the adopted budget for a variety of reasons. State Statutes require that official modifications to the adopted budget be approved by a two-thirds majority of the Board of Education and that there be a publication of a Class 1 notice within 10 days of approval. This document identifies budget modifications to the 2021-2022 budget delineated by fund and project.

The majority of these changes are the result of carryover notifications determined to be available for various grants/programs after the budget was formally adopted. Other grant awards (e.g. Education Foundation, mini-grants) were also received after the adoption of the budget. These grant awards conform to existing Board policy and have been previously shared with the Board of Education through the approval of the grant.

Since State Statutes authorize the budget to be adopted by function; the administration also requests approval of additional budget modifications that did not add or subtract dollars to the overall budget, but may have changed the function or purpose of the funding.

Attachment A is a copy of the Notice of Change in Adopted Budget in the proper State approved format that will need to be published in the Kenosha News after the Board has approved these budget modifications.

The administration requests that the School Board approve this report and that the attached Class 1 notice be published within 10 days of the official Board adoption."

Mr. Battle moved to approve the Change in the Fiscal Year 2021-22 Adopted Budget and that the attached Class 1 notice be published within 10 days of the official Board adoption. Mrs. Modder seconded the motion. Unanimously approved.

Mr. Hamdan presented the 2021-2022 Budget Carryovers to the 2022-2023 Budget submitted by Mrs. Salo, Accounting Manager; Mr. Hamdan; and Dr. Weiss, excerpt follow:

“Historically, Kenosha Unified School District (KUSD) has prohibited the automatic carryover of unutilized budget authority from one fiscal year to the next. At the August 9, 2000 meeting of the School Board, it was unanimously approved to discontinue the practice of automatic site carryovers. Currently, carryover authority is only approved if required by an outside agency or if it is specifically approved by the Board on an exception basis.

Site requested carryovers require a pre-approved specific purpose before they are brought forward for Board consideration. The Department of Teaching and Learning has requested to carry over $1,987,000 which represents the full curriculum adoption budget that was allocated in FY 22 but was not spent. This amount has historically been a standing annual amount allocated for this specific purpose. Moving forward, the administration has recommended the implementation of an as-needed approach where Teaching and Learning would identify and request specific funding for the upcoming year rather than a standing annual allocation.

Certain funding that is provided to our district is required by the Department of Public Instruction (DPI) to be carried over into the following fiscal year if all the funds were not spent on the designated purpose within the fiscal year in which they were received. Such is the case for the $128,259 balance of Common School Library Funds and the $152,064 balance of Career and Technical Education Incentives.

During the previous school year, several schools/departments received cash donations or mini-grants from outside organizations, most notably from the Education Foundation of Kenosha (EFK). Some of the donated funds totaling $290,896 were not completely spent by the end of the school year; therefore these funds are carried over to the next year to be spent on the programs as intended by the donors.

Through an agreement amongst Finance, Athletics, and Facilities, we have arranged to earmark rental revenue generated at our various athletic fields so that it will be used specifically for the maintenance and upkeep of those fields. The $45,639 balance of these funds is recommended for carryover so that it can be used for the intended purpose.

Starting in the 2018-19 fiscal year, KUSD began transitioning some accounts previously held in Student Activities (Fund 60) to the General Fund (Fund10) due to their co-curricular purposes. These accounts included a variety of revenue sources including ticket sale proceeds. The $82,501 balance of these funds is recommended for carryover so that it can be used for the intended purpose.

KUSD instrumentality charter schools are allowed access to any of their earmarked and unspent general fund dollars, as stipulated in their contracts (charters) with the district. This is necessitated due to the unique funding of the schools, the responsibility they have for their respective budgets, and their responsibility for future major maintenance issues or technology replacements not funded by the district. Starting in the fiscal year 2012-2013, charter school carryovers were accounted for as assigned portions of the general fund balance rather than being added as additional amounts in expense budgets as they used to be. This method provides for a more accurate year-to-year budgeting while preserving the charter school’s access to surplus funds. The schedule at the bottom of Attachment A shows the total balance in the charter school fund balance reserve accounts as $3,706,743 as of June 30, 2022.

The administration requests that the School Board approve this report so that these carryover funds can be incorporated into the adopted 2022-2023 budget."

Mrs. Modder moved to approve the 2021-2022 Budget Carryovers so that the carryover funds can be incorporated into the adopted 2022-2023 budget. Ms. Stevens seconded the motion. Unanimously approved.

Mr. Hamdan presented the Formal Adoption of the 2022-23 Budget submitted by himself, Mrs. Salo, and Dr. Weiss, excerpts follow:

"The public hearing on the 2022-23 budget and the annual meeting of district electors were held on September 20, 2022, in the auditorium of Indian Trail High School and Academy. In an advisory vote at the annual meeting of district electors, stakeholders in attendance voted to set the total tax levy at the maximum allowed by law, which was initially projected to be a total of $84,903,530. At the time of the annual meeting, it is important to note that key variables in the budgeting process were not finalized, therefore conservative estimates were included.

Since the public hearing and the annual meeting, the administration has updated the budget to reflect key components such as student membership, equalized property valuations, certified state aid, tax levies, and detailed staffing costs (salary and benefits).

We continue to experience a decline in our total student full-time equivalents (FTE) which drives our revenue limit authority. While 3rd Friday enrollment counts came in better than originally expected, once converted into membership FTE, we still experienced an overall loss of 159 FTE for revenue limit authority purposes. Our continual declining enrollment status triggers additional temporary (non-recurring) revenue limit exemptions that are meant to buy us time and provide temporary budget relief as we prepare to make adjustments to our operations. These exemptions include a hold-harmless amount of $5,051,237 and a declining enrollment exemption amount of $5,051,241.

In the official October 15th certification, our general state aid decreased by $1,462,601 or -0.98% as compared to last year. Our total state aid that impacts tax levy decreased by a total of $1,310,273 or -0.87%.

The 2022-23 general fund (10) is being presented as an unbalanced budget in which expenditures are projected to exceed revenues by $367,661 if all budget authority is fully exercised. Unlike previous years, the budget is not in a positive position with unallocated funds that could be used to absorb the carryover spending authority requests submitted to the Board for consideration. Any approved carryover authority will increase the budgeted expenditures and increase the deficit or difference between expected revenues and expenditures.

However, approximately $2.6 MM of Federal stimulus funding was received in FY 2021-22 via Governor Evers' Promise funds (one-time funding of $134 per pupil to help offset the lack of inflationary increases in the 2021-2023 State Biennial Budget). These funds were not used last year and they have been retained in fund balance reserves for budget stabilization in FY 2022- 23. Any deficit spending up to the $2.6 MM threshold would be considered a planned one-time use of reserves.

For the 2022-23 fiscal year, Governor Evers has directed another round of stimulus funds to be distributed to schools in the form of a $91 per pupil allocation that equates to about $1.8 MM for KUSD which has also been built into this budget.

The proposed tax levy for the general fund (10) is the maximum amount allowed within state law without passing a referendum. The overall 6.24% decrease in total tax levy equates to $5,596,745 less local property tax dollars needed for the Kenosha Unified School District as compared to the previous year. The decrease in tax levy is directly correlated to changes in state aid and decreasing revenue limit authority driven by declining enrollment in the general fund.

The total mill rate (tax per $1,000 of equalized property valuation) is $6.24, a 19.85% decrease as compared to the prior year's rate of $7.78. This decrease is the result of changes in both tax levies and equalized property values in our district. Our equalized property value increased by 16.98% from last year allowing the reduced tax levy to be spread over an even larger tax base. This results in a more dramatic change in the mill rate. While the State average increase in equalized property values is about 14%, the district also experienced a bump in property values due to the closing of the Pleasant Prairie Tax Increment District (TID) #2. This tax levy scenario and a historical view of the District's equalized property values, tax levies, and mill rates are shown in Attachment A.

It is requested that the Board of Education accept the following recommendations:

1. Formally adopt the District's 2022-2023 budget using the accompanying budget adoption motion (Attachment B).

2. Direct the administration to prepare a class one legal notice to be published publicly within ten days of the adoption (Attachment C).

3. Approve the property tax levy to be collected from the municipalities within the school district in the amount of $70,288,237 for the general fund, $12,311,491 for the debt service fund, and $1,500,000 for the community service fund. The Board must approve levy amounts on or before November 1st each year, per Wis. Stats. 120.12 (3)(a).

4. Direct the district clerk to certify and deliver the Board approved tax levy to the clerk of each municipality on or before November 10, 2022. "

Mrs. Modder moved that the 2022-2023 budget for the Kenosha Unified School District, as presented, for all funds showing expenditures, other revenues, and tax levies in summary be adopted as set forth below and in the accompanying format required by the Wisconsin Department of Public Instruction (see Attachment C); that administration prepare a class one legal notice to be published publicly within ten days of the adoption (Attachment C); that the Board approve the property tax levy to be collected from the municipalities within the school district in the amount of $70,288,237 for the general fund, $12,311,491 for the debt service fund, and $1,500,000 for the community service fund as the Board must approve levy amounts on or before November 1st each year, per Wis. Stats. 120.12 (3)(a), and that the district clerk certify and deliver the Board approved tax levy to the clerk of each municipality on or before November 10, 2022. Mrs. Stevens seconded the motion. Unanimously approved.

Mrs. Modder presented Resolution 403 – American Education Week: November 14-18, 2022 which read as follows:

“WHEREAS, American Education Week is designated to celebrate and honor the individuals who are dedicated to ensuring every child receives a quality education; and

WHEREAS, collaborative sponsors include the U.S. Department of Education and national organizations, including the American Association of School Administrators, the American Federation of Teachers, the American Legion, the American Legion Auxiliary, the American School Counselor Association, the Council of Chief State School Officers, the National Association of State Boards of Education, the National Association of Elementary School Principals, the National Association of Secondary School Principals, the National Education Association, National PTA, the National School Boards Association, and the National School Public Relations Association; and

WHEREAS, public schools are the backbone of our democracy, providing young people with the tools they need to maintain our nation's precious values of freedom, civility and equality; and

WHEREAS, all students, parents, employees, volunteers and stakeholders share responsibility for promoting and maintaining a nurturing, safe academic environment in which every student shall be provided excellent, challenging learning opportunities and experiences that prepare them for success; and

WHEREAS, all Kenosha Unified staff work tirelessly to serve our children and community with great care and professionalism; and

WHEREAS, our schools encourage the bringing together of children, families, educators, volunteers, business leaders and elected officials in a common enterprise that offers exceptional opportunities in academics and extracurricular activities to provide students with the skills needed to grow and succeed in a global society.

NOW, THEREFORE, be it resolved that Kenosha Unified School District does hereby proclaim November 14-18, 2022, as the annual observance of American Education Week.

BE IT FURTHER RESOLVED, that a true copy of this resolution be spread upon the official minutes of the Board of Education.”

Mrs. Modder moved to approve Resolution 403 – American Education Week: November 14-18, 2022. Mr. Price seconded the motion. Unanimously approved.

Mrs. Schmaling presented Resolution 404 – National Native American Heritage Month 2022 which read as follows:

“WHEREAS, National Native American Heritage Month is celebrated from November 1 through November 30 as a way to consider and recognize the contributions of Native Americans to the history of the United States of America; and

WHEREAS, Native Americans are descendants of the original, indigenous inhabitants of what is now the United States; and

WHEREAS, Native Americans have made important contributions to the United States and the rest of the world as business owners, artists, teachers, writers, members of our Armed Forces, and much more; and

WHEREAS, Their contributions to our society are cause for celebration and appreciation; and

WHEREAS, The month is a time dedicated to celebrating their rich and diverse cultures, traditions, and histories while acknowledging the importance of their contributions; and

WHEREAS, National Native American Heritage Month is an opportune time to educate students about tribes, raise a general awareness about the unique challenges Native Americans have faced both historically and in the present, and the ways in which tribal citizens have worked to conquer these challenges; and

WHEREAS, corresponding school activities held in November, as well as throughout the school year, will educate students about Native American cultures, traditions and contributions that have impacted business, law, education, politics, science, the arts and more.

NOW, THEREFORE, BE IT RESOLVED that Kenosha Unified School District’s Board of Education does hereby adopt this resolution to proclaim November as National Native American Heritage Month.

BE IT FURTHER RESOLVED, that a true copy of this resolution be spread upon the official minutes of the Board of Education.”

Mrs. Schmaling moved to approve Resolution 404 – National Native American Heritage Month 2022. Ms. Stevens seconded the motion. Unanimously approved.

Ms. Stevens presented the Donations to the District.

Ms. Stevens moved to approve the donations to the district as presented. Mr. Price seconded the motion. Unanimously approved.

Meeting adjourned at 9:17 P.M.

Stacy Stephens
School Board Secretary

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Kenosha Unified School District
Kenosha, Wisconsin
Summary of Cash Receipts and Disbursements
November 15, 2022

CASH RECEIPTS	reference	total
October 2022 Wire Transfers-In, to Johnson Bank from:		
WI Department of Public Instruction	*state aids register receipts*	$ 760,559.88
Johnson Bank	*account interest*	433.10
Bankcard Services (MyLunchMoney.com)	*food services credit card receipts (net of fees)*	121,780.48
Bank (Infinite Campus)	*district web store receipts (net of fees)*	47,876.44
Retired & Active Leave Benefit Participants	*premium reimbursements*	46,165.80
HHS	*head start grant*	253,903.33
Various Sources	*small miscellaneous grants / refunds / rebates*	8,526.52
Total Incoming Wire Transfers		1,239,245.55
October 2022 Deposits to Johnson Bank - All Funds:		
General operating and food services receipts	*(excluding credit cards)*	203,443.37
TOTAL OCTOBER CASH RECEIPTS		$ 1,442,688.92

CASH DISBURSEMENTS	reference	total
October 2022 Wire Transfers-Out, from Johnson Bank to:		
Payroll & Benefit wires		
Individual Employee Bank Accounts	*net payrolls by EFT (net of reversals)*	$ 8,240,776.68
WI Department of Revenue	*state payroll taxes*	581,032.98
WI Department of Revenue	*state wage attachments*	3,502.73
IRS	*federal payroll taxes*	2,858,198.47
Delta Dental	*dental insurance premiums*	227,047.31
Diversified Benefits Services	*flexible spending account claims*	7,271.97
Employee Trust Funds	*wisconsin retirement system*	1,554,704.80
NVA	*vision insurance premiums*	19,932.36
Aflac	*insurance premiums*	42,474.86
Optum	*HSA*	280,778.22
Various	*TSA payments*	343,813.51
Subtotal		14,159,533.89
General Operating Wires		
US Bank	*purchasing card payment-individuals*	174,487.53 *
Aegis	*workers' compensation payment*	200,000.00
Kenosha Area Business Alliance	*LakeView lease payment*	17,664.63
United Healthcare	*health insurance premiums*	3,891,855.07
Subtotal		4,284,007.23
Total Outgoing Wire Transfers		$ 18,443,541.12
October 2022 Check Registers - All Funds:		
Net payrolls by paper check	*Register# 01020DP, 01021 DP*	$ 7,327.06
General operating and food services	*Check# 616729 thru Check# 617588 (net of void batches)*	4,140,128.33
Total Check Registers		$ 4,147,455.39
TOTAL OCTOBER CASH DISBURSEMENTS		$ 22,590,996.51

**See attached supplemental report for purchasing card transaction information*

KUSD Purchasing Card Program - Individual Cardholders

Transaction Summary by Merchant

Billing Cycle Ending October 17, 2022

Merhant Name	Total	
3654 INTERSTATE	$	12,106.07
RESTAURANTS & CATERING	$	11,072.39
MENARDS KENOSHA WI	$	8,079.81
MARK'S PLUMBING PARTS	$	7,516.25
HAJOCA KENOSHA PC354	$	7,185.69
ULINE *SHIP SUPPLIES	$	7,093.62
NORTHERN MECHANICAL...	$	5,512.00
IC* INSTACART	$	4,227.23
VEHICLE MAINT. & FUEL	$	3,412.06
CNK*CINEMARK HQ 001	$	3,289.50
CHESTER ELECTRONIC SUPPLY	$	2,583.84
GRANITE VALLEY FOREST PRO	$	2,396.02
AIRLINE	$	2,305.20
JOHNSTONE SUPPLY - RACINE	$	2,296.71
HOTEL	$	2,126.28
L AND S ELECTRIC INC	$	2,046.00
JOHNSON CONTROLS SS	$	1,985.08
LAKESIDE INTERNATIONAL	$	1,969.90
THE LINE UP	$	1,885.83
MILWAUKEE COUNTY ZOO	$	1,771.25
WASBO FOUNDATION	$	1,660.00
MUSICFOLDER.COM - USD	$	1,575.44
APPLE HOLLER	$	1,542.00
TRUGREEN *LOCKBOX	$	1,535.10
MARZANO RESEARCH LAB	$	1,418.00
BROADWAY MEDIA	$	1,346.25
SQ *GET "ALL-IN" LLC	$	1,327.00
EUROFINS SF ANALYTICAL LA	$	1,275.22
LOWES #02560*	$	1,259.92
OTC BRANDS INC	$	1,226.65
GRAINGER	$	1,205.09
HALLMAN LINDSAY PAINTS -	$	1,177.59
THE ASSOCIATED COLLEGIATE	$	1,144.49
AMZN MKTP US*1U7IW3CW0	$	1,140.00
AMZN MKTP US*1U5UQ5O90	$	1,136.76
JC LICHT - 1290 - KENOSHA	$	1,110.54
HUDL	$	1,098.00
FOUNDATION BLDG 045	$	1,073.36
MONSTER JANITORIAL LLC	$	1,051.55
POWER EQUIPMENT DIRECT	$	1,027.99
HIGHWAY C SERVICE INC	$	1,027.26
CLEANFREAK	$	961.65
ZORO TOOLS INC	$	944.64
MENARDS RACINE WI	$	922.43
FACEBK DUXFKFX782	$	900.00
HALOGEN SUPPLY COMPANY IN	$	892.82
RSCHOOL TODAY	$	890.00

KUSD Purchasing Card Program - Individual Cardholders

Transaction Summary by Merchant

Billing Cycle Ending October 17, 2022

Merhant Name	Total	
HAWK'S VIEW GOLF CLUB	$	880.00
FIRST SUPPLY KENOSHA	$	851.79
RAPID DEFENSE	$	844.00
CUSTOMINK LLC	$	834.72
BEST BUY 00011916	$	828.94
COSPACES	$	825.00
AZTEC FENCE COMPANY	$	816.00
WWW SCHOOLMATE COM	$	806.25
MILWAUKEE SYMPHONY ORCHES	$	783.00
SUNNYSIDE FLORIST OF KENO	$	783.00
SPORTS UNLIMITED, INC.	$	779.80
SAMS CLUB #6331	$	772.34
VACLAND	$	710.30
HOERNEL KEY SHOP	$	705.00
CDW GOVT #DB70538	$	700.89
CARNES COMPANY	$	700.48
LINCOLN CONTRACTORS SUPPL	$	690.76
VAISALA, INC., BOULDER	$	682.00
STERICYCLE	$	669.13
KIMBALL MIDWEST PAYEEZY	$	665.32
SHERWIN WILLIAMS 703481	$	658.86
SAN-A-CARE	$	655.04
CROWN EQUIPMENT CORP	$	648.83
VIKING ELECTRIC-MILWAUKEE	$	645.96
TOOLUP.COM	$	641.96
WM SUPERCENTER #1167	$	629.19
LAMINATION DEPOT INC	$	624.52
USPS PO 5642800260	$	589.19
MAXON EQUIPMENT INC.	$	577.77
GUIDED READERS	$	567.00
PFG*PROFORMA	$	564.00
IN *GOOD ARMSTRONG TRAINI	$	550.00
NATL ARCHERY SCHOOLS ECOM	$	548.00
DRAMATISTS PLAY SERV	$	538.00
AMAZON.COM*1U1CZ4GL0	$	535.00
MCMASTER-CARR	$	530.24
IDVILLE	$	517.86
WAL-MART #1167	$	489.46
AMAZON.COM*148VL40I1 AMZN	$	474.50
NBX*SLATE 6082410300	$	470.00
WISCONSIN ASSOCIATION OF	$	466.00
SAMSCLUB #6331	$	451.69
FARM & FLEET STURTEVANT	$	449.00
BZOO - WEBSITE ADMISSI	$	441.00
CONCORD THEATRICALS CORP.	$	440.15
SPIRAL BINDING LLC	$	435.04
JANSSEN SPORTS LDRSHP.	$	425.00

KUSD Purchasing Card Program - Individual Cardholders

Transaction Summary by Merchant

Billing Cycle Ending October 17, 2022

Merhant Name	Total	
DOLLAR TREE, INC.	$	401.97
IN *ADS DISPOSAL	$	400.00
SP ORTHOBRACING.COM	$	387.76
AMZN MKTP US*142HK1TN2	$	384.29
SQ *KENOSHA HISTORY CENTE	$	368.00
UNIFILLER SYSTEMS INC.	$	363.64
BLS*NOVISIGN LTD	$	360.00
PICK N SAVE #871	$	347.91
FESTIVAL FOODS	$	339.15
AMAZON.COM*149RZ8PW1	$	337.59
AMAZON.COM*1K8T497S0 AMZN	$	331.60
WALMART.COM AA	$	325.18
WIAIMH	$	323.00
SP DEFENSEONASTRING	$	321.98
COSTCO WHSE #1198	$	321.83
LOWES #00907*	$	309.76
PRESS TIME CLEANERS	$	300.00
SMARTSIGN	$	299.15
AMZN MKTP US*1M97F16P0	$	284.00
HOMEDEPOT.COM	$	283.49
IC* COSTCO BY INSTACAR	$	279.88
AMAZON.COM*1U4JO4270	$	278.27
IN *LITTLE SPONGES	$	275.00
4IMPRINT, INC	$	262.65
INSTACART	$	254.39
OFFICEMAX/DEPOT 6358	$	250.25
NASSP PRODUCT & SERVICE	$	250.00
U OF I ONLINE PAYMENT	$	250.00
AMZN MKTP US*1M9JQ4WQ1	$	247.78
AMZN MKTP US*1U5F11N21	$	247.67
USPS.COM POSTAL STORE	$	246.30
1000BULBS.COM	$	239.89
SMUGMUG.COM	$	237.38
LEARNING FORWARD (LF)	$	237.00
SMORE.COM - EDUCATOR	$	237.00
MENARDS 3327	$	236.25
VISTAPRINT	$	226.80
FASTENAL COMPANY 01WIKEN	$	223.71
PODS 9/100	$	221.40
ESTESROCKETS.COM	$	219.95
MICHAELS STORES 9192	$	219.78
ROCKLER	$	219.58
CREATEFORLESS.COM	$	215.84
SIGNUPGENIUS	$	215.78
SHERWIN WILLIAMS 703180	$	215.34
SCHOLASTIC, INC.	$	213.75
AMZN MKTP US*1K0ER22S2	$	213.02

KUSD Purchasing Card Program - Individual Cardholders

Transaction Summary by Merchant

Billing Cycle Ending October 17, 2022

Merhant Name	Total	
SQ *DJ FELIX ENTERTAINMEN	$	211.00
TOWN & COUNTRY GLASS	$	205.31
V BELT GLOBAL SUPPLY	$	204.97
SAMSCLUB.COM	$	200.16
GANDER #637	$	199.97
AMZN MKTP US*1K9RP33X1	$	199.00
CHRISTOPHER R GREEN SR	$	199.00
THE WEBSTAURANT STORE INC	$	197.51
24HOURWRISTBANDS.COM	$	197.20
MID STATE EQUIP SALEM 010	$	196.98
SCHAUER POWER CENTER	$	191.89
SQ *THE FUNKIN FARM	$	188.00
PARTY CITY 5174	$	185.90
AUER STEEL MILWAULKEE	$	180.95
LIFESPORT - KENOSHA POS	$	180.00
AMAZON.COM*1U2GL3FH1 AMZN	$	179.00
AMAZON.COM*1U0SU0KE0 AMZN	$	172.98
HOLLAND SUPPLY INC	$	171.92
AMZN MKTP US*1K1TH2GR2	$	167.26
SPECTRUM	$	161.46
USA CLEAN BY JON-DON	$	160.97
DOLLAR TREE	$	159.46
GFS STORE #1919	$	156.55
YAHOO SMALL BUSINESS	$	155.76
AMZN MKTP US*1K6QY3U20	$	142.90
AMZN MKTP US*145Y89O40	$	141.36
4PROMOS LLC	$	138.65
FLOCABULARY	$	138.00
FLUID HANDLING INC	$	134.41
JON DON ECOMM #999	$	132.43
CENTRAL SAW & MOWER	$	128.66
CAREERSAFE ONLINE	$	128.00
WIKKI STIX CORP	$	123.15
KENOSHA AREA BUSINESS	$	115.00
D W DAVIES & CO INC	$	112.68
MICHAELS #9490	$	111.57
DECKER EQUIPMENT	$	108.85
SHIFFLEREQUIP.COM	$	107.83
AMZN MKTP US*1M17U7ZT0	$	103.94
DESIGN AIR	$	101.13
SOUTHEASTERN EQUIPMENT &	$	100.20
SKILLSUSA ORG	$	100.00
EDULASTIC SUBSCRIPTION	$	100.00
PADLET* PADLET SOFTWAR	$	99.99
DELTAMATH.COM	$	95.00
PRAIRIE SIDE ACE HDWR	$	95.00
VYRON CORPORATION	$	90.00

KUSD Purchasing Card Program - Individual Cardholders

Transaction Summary by Merchant

Billing Cycle Ending October 17, 2022

Merhant Name	Total	
JADE LEARNING LLC	$	90.00
ASCD MEMBERSHIP	$	89.00
AMAZON.COM*1K5SI9482	$	85.94
NATIONAL COUNCIL OF SUPER	$	85.00
AMZN MKTP US*1U5OU7590	$	82.20
AMZN MKTP US*1M92K2GE0	$	79.93
CABLE TIES PLUS	$	78.73
FIELDPRINT INC	$	78.00
HOBBY LOBBY #350	$	75.69
MICHAELS STORES 5193	$	73.99
AMZN MKTP US*1M9OL46Q0 AM	$	72.93
MAILCHIMP	$	69.99
AMZN MKTP US*1K1US5J61	$	68.20
DOLLARTREE	$	67.00
FAMILY DOLLAR #1761	$	65.55
AMERICAN RED CROSS	$	65.00
SCREENCASTIFY UNLIMITE	$	65.00
SP INSTRUCTIONAL COA	$	64.11
APPLE.COM/BILL	$	63.29
ADOBE STOCK	$	63.28
SP FERRY-MORSE GARDN	$	62.72
BUILDING CONTROLS & SOLUT	$	62.67
JOHN POWLESS TENNIS CENTE	$	60.00
CHARTHOUSE	$	51.68
GFS STORE #1923	$	50.98
AMZN MKTP US*1U9T91SE0 AM	$	49.95
EDPUZZLE PRO TEACHER	$	49.00
AMZN MKTP US*1M2LI69Y2	$	48.10
JANESVILLE COUNTRY CLUB I	$	47.48
AMZN MKTP US*1K9WU9WG0	$	42.26
THE MATH LEARNING CENTER	$	40.00
AMZN MKTP US*145V29YZ1	$	39.99
AMZN MKTP US*1K9ZR1RW0	$	39.96
AMZN MKTP US*HT49B84M2	$	39.09
SMK*WUFOO.COM CHARGE	$	39.00
DDPYOGA	$	36.91
AMZN MKTP US*1K3B06PO2	$	35.94
BLUE HOUSE BOOKS	$	35.17
TENNISREPORTING	$	35.00
AMZN MKTP US*1K9TD9GK2	$	34.69
AMAZON.COM*1M9Z637A1 AMZN	$	33.00
AMZN MKTP US*1K1489ZM0	$	32.54
ZOOM.US 888-799-9666	$	30.80
FACEBK D85MTHK882	$	29.01
STATSMEDIC.COM	$	29.00
WPY*CONTINUING EDUCATION	$	29.00
AMZN MKTP US*1K40K1KM0	$	27.92

KUSD Purchasing Card Program - Individual Cardholders

Transaction Summary by Merchant

Billing Cycle Ending October 17, 2022

Merhant Name	Total	
IVES GROVE GOLF LINKS	$	27.00
AMZN MKTP US*1U6C98PL1	$	25.47
QM QUALITY MATTERS	$	25.00
MOVEMENT OVER MAXES	$	24.99
AMZN MKTP US*1M2869TN0	$	23.99
WALGREENS #3617	$	23.15
AMZN MKTP US*1M4MA0T92	$	23.00
TARGET 00027656	$	22.37
WALMART.COM	$	20.66
AMZN MKTP US*1M1PV5HJ0	$	18.84
PICK N SAVE #874	$	17.57
AMZN MKTP US*HT3QX8KH0	$	16.78
MEIJER # 284	$	15.99
MENARDS WEST ALLIS WI	$	15.61
MUSKEGO LAKES COUNTRY CLU	$	14.50
AMAZON.COM*HT3RJ8FY0	$	13.06
RANGE TIME	$	13.00
SPOTIFY USA	$	10.54
AMZN MKTP US*HT24T8AI1	$	9.99
EXPEDIA 72403942004681	$	9.13
AMZN MKTP US*HT1F55960	$	8.39
UPS*29W93FRFOD7	$	7.00
MEADOWBROOK COUNTRY CLUB	$	3.00
SUCCESS BY DESIGN INC	$	(107.27)
AP EXAM TOTALREG	$	(291.00)
AMZN MKTP US	$	(495.45)
AMAZON.COM AMZN.COM/BILL	$	(521.84)
US Bank Purchasing Card Payment - Individuals	$	174,487.53

KENOSHA UNIFIED SCHOOL DISTRICT
Kenosha, Wisconsin

November 15, 2022

Administrative Recommendation

It is recommended that the October 2022 cash receipt deposits totaling $203,443.37, and cash receipt wire transfers-in totaling $1,239,245.55, be approved.

Check numbers 616729 through 617588 (net of voided batches) totaling $4,140,128.33, and general operating wire transfers-out totaling $4,284,007.23, are recommended for approval as the payments made are within budgeted allocations for the respective programs and projects.

It is recommended that the October 2022 net payroll and benefit EFT batches totaling $14,159,533.89, and net payroll check batches totaling $7,327.06, be approved.

Dr. Jeffrey Weiss
Superintendent of Schools

Tarik Hamdan
Chief Financial Officer

Lisa M. Salo, CPA
Accounting Manager

KENOSHA UNIFIED SCHOOL DISTRICT
Kenosha, Wisconsin

November 15, 2022

REPORT ON CONTRACT IN AGGREGATE OF $50,000

School Board Policy 3420 requires that "all contracts and renewals of contracts in aggregate of $50,000 in a fiscal year shall be approved by the School Board except in the event of an emergency as determined and reported to the School Board monthly by the Purchasing Agent."

The following contract/agreement has not been added to the Contract Management Database and is being presented for board approval:

VENDOR	PROGRAM/PRODUCT	AMOUNT
EMC2	Digital platform for Engagement and Professional Learning	$139,500.00 (Title IV Funding)

The Purchase/Contract Rationale is provided in Appendix A. The quote from EMC2 is provided in Attachment A. The Letter of Ownership from EMC2 is provided in Attachment B.

Recommendation

Administration recommends that the Board of Education approve the $139,500.00 contract with EMC2 for a digital platform to provide teachers with student engagement resources and on-demand access to professional learning.

Dr. Jeffrey Weiss
Superintendent of Schools

Julie Housaman
Chief Academic Officer

Tarik Hamdan
Chief Financial Officer

Robert Hofer
Purchasing Agent

Che Kearby
Coordinator of Secondary English
and Social Studies

Jen Lawler
Coordinator of Secondary Math
and Science

101722—2

Kenosha Unified School District

3600 52ND ST., KENOSHA, WI 53144
P 262-359-6300 • F 262-359-7672

APPENDIX A

PURCHASE/CONTRACT RATIONALE

Per School Board Policy 3420, please complete the following to be attached to your purchase order/contract. Additional information may be required and presented before the District's School Board for approval. Your submission must allow for adequate time for the Board to approve.

Vendor: EMC2

Purchased Good/Program: Digital Platform for Engagement and Professional Learning

Start Date/Date Needed: 11/16/2022

1. PURPOSE – What is the purpose of the proposed purchase?

EMC2 Learning will provide the middle and high school instructional staff with access to the Creative Corps PD Platform. This service connects hundreds of vetted, research backed strategies for educators to design student centered instruction using a feedback based approach and high quality engagement strategies. Middle and high school teachers embarked on a yearlong professional learning focus on high quality instruction during welcome back week in August. Teaching & Learning is requesting to contract with EMC2 in order to: work in collaboration with T&L in the design and implementation of monthly professional learning sessions, provide access to the Creative Corps Platform that offers on-demand virtual professional learning as well as a library of active engagement resources. The resources compliment district curriculum and support the design of engaging lessons that are available to teachers 24/7.

2. FUNDING – What is the total cost of purchase and the funding source?

Total cost: $139,500.00 (Attachment A: EMC2 Quote)

Funding source: Title IV

3. REQUEST FOR PROPOSAL (RFP) – indicate if an RFP has been completed

YES ☐ NO ☑ If no, please request an RFP packet
(Attachment B: Sole Source Document)

4. EDUCATIONAL OUTCOME – What is the educational outcome of this purchase?

The educational outcome will be engaging lessons which focus on critical thinking, communication, collaboration, and creativity while preparing our students for post high school success. Students will develop the skills necessary to succeed in college, technical school, or the workplace. Increased engagement in the classroom will lead to improved literacy and math scores which will help provide students with choices after high school. Teachers will be provided ample opportunity to receive coaching and space to collaborate with colleagues while designing rigorous lessons using the district curriculum. Success will be measured using test scores, feedback and walk through tools.

5. START DATE – When is the anticipated start date?

November 16, 2022

Your response does not establish approval of either a contract or a purchase order.

Appropriate Leadership Signature Julie Housaman Date Oct. 17, 2022

APPENDIX A
ATTACHMENT A

EMC² LEARNING
ENGAGEMENT AT THE SPEED OF LIFE

EMC² Learning
PO BOX 141, Port Washington
WI 53074 United States
www.EMC2Learning.com

1. Pursuant to this Order Form, Subscriber shall receive the Services from EMC² Learning, LLC. that are described in the table provided on page 2 of this agreement. The first table ("Non Discounted Rate") reflects the total non-discounted value for the Services to be provided by EMC² Learning. This Order Form expands on Services rendered to the **Kenosha Unified School District** by EMC² Learning during the 2021-2022 Academic Year – thus securing additional Services for the 2022-2023 Academic Year at a discounted, bulk rate price such fees shall be paid according to revised pricing table ("Discounted Subscriber Fees"). Subscriber shall be responsible for paying to EMC² Learning the discounted rate of the total fees indicated in the second table below.

2. Subscriber agrees that EMC² Learning may disclose certain product, services, and website usage data ("Subscriber Data") to the public for future resource development and promotional purposes. The Subscriber Data shall be categories of de-identified, anonymized, and/or aggregated data, including, but not limited to, affidavits of individual teachers, screenshots and/or animated images of modified templates created through resources available in the EMC² Learning library, aggregate count of teacher site usage, aggregate count of administrator log-ins, school lists based on number of students using the Services, and top resources used by teachers. The Subscriber Data shall not include confidential information, personally identifiable information or student education records as defined under applicable state or federal laws.

3. The Subscriber shall be responsible for paying to EMC² Learning the remaining portion of the total fees as set forth in further detail below (the "Discounted Subscriber Fees").

4. All resources require an active partnership with EMC² Learning. After the contract expires, the district and its employees cannot use any of the saved lessons or activities. An active contract is required in order to use these resources and any iterations of them.

5. Subscriber Fees: All Fees are non-cancellable and are due and payable NET 60

EMC² LEARNING
ENGAGEMENT AT THE SPEED OF LIFE

Services To Be Rendered by EMC² Learning During 2022-2023 Academic Year

(Discounted Subscriber Fees)

SERVICES	START DATE	END DATE	QTY - UNIT OF MEASURE	PRICE	TOTAL
Building Wide Access with One Year Creative Corps Subscription to EMC2Learning.com Resource Library & the Sustainability Plan for district level	7/1/2022	6/30/2023	11 Buildings (approx. 600 teacher accounts)	83.17	$139,500
					$139,500.00

EMC² LEARNING
ENGAGEMENT AT THE SPEED OF LIFE

This Order Form constitutes the entire agreement between Subscriber and EMC² Learning for the Services to be rendered in the 2022-2023 Academic Year. By signing below, the Subscriber and EMC² Learning agree to be bound by this Order Form and the Standard Terms as of the date of the last signature below.

KENOSHA UNIFIED SCHOOL DISTRICT	EMC² LEARNING
BY: ____________________ *(Signature Required)*	BY: ____________________ *(Signature Required)*
TITLE: ____________________	TITLE: ____________________
PRINTED NAME: ____________________	PRINTED NAME: ____________________
DATE: ____________________	DATE: ____________________

EMC² LEARNING
ENGAGEMENT AT THE SPEED OF LIFE

EMC2 Learning
PO Box 141
Port Washington, WI 53074

September 7, 2022

Re: Letter of Ownership

To Whom it May Concern,

This Letter of Ownership is to confirm that EMC² Learning, a Wisconsin based company, is the proprietary and copyright owner of the product(s) on our platform. Creative Corps Membership level that is being purchased by KENOSHA UNIFIED SCHOOL DISTRICT for their Middle and High schools and the creator or authorized licensee of any of the content contained therein or on which such service is based. EMC² Learning is the sole provider of Creative Corps PD Platform. This is a unique and complex service that connects hundreds of vetted, researched backed, strategies for educators to create more student centered classrooms through a feedback based approach using sound engagement strategies. An active and up to date district license is required to use any of our resources. Any and all rights to manufacture, license, distribute, market or sell the Products remain and are reserved in and to EMC² Learning.

If you have any questions, please contact
Michael Matera

By email at: connect@emc2learning.com

Sincerely,

Michael Matera
CEO of EMC2 Learning

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KENOSHA UNIFIED SCHOOL DISTRICT
Kenosha, Wisconsin

November 15, 2022

COURSE CHANGE PROPOSAL FOR HIGH SCHOOL MATHEMATICS AND INFORMATION TECHNOLOGY

Background

In May 2022 The College Board announced the launch of Advanced Placement Precalculus that will be available to students in the 2023-24 school year. The Advanced Placement Program convened college faculty to build a precalculus course that invites a diverse group of students to prepare for college mathematics, encourages more students to complete four years of mathematics in high school, and improves student readiness to succeed in science, technology, engineering, and mathematics courses and majors in college. The proposed high school mathematics course change enhances the mathematics course options that were approved by the Board in May 2022.

LakeView Technology Academy, under the guidance of Gateway Technical College, is proposing the addition of three new courses for the 2023-24 school year: Mobile Game Programming, Game Programming Technologies, and Front-End Development with Angular. With the addition of these three courses, LakeView Technology graduates will have the opportunity to graduate with two Gateway certifications in programming: Full Stack Web Developer and Game Programming.

Course Change Proposal Request

The Office of Teaching and Learning is proposing, for Board of Education approval, the addition of Advanced Placement Precalculus, Mobile Game Programming, Game Programming Technologies, and Front-End Development with Angular to the 2023-24 course catalog.

COURSE	SCHOOLS	APPENDIX
Advanced Placement Precalculus	Bradford, Harborside, Indian Trail, LakeView, and Tremper	A
Mobile Game Programming	LakeView Technology Academy	B
Game Programming Technologies	LakeView Technology Academy	C
Front-End Development with Angular	LakeView Technology Academy	D

Recommendation

Administration recommends that the Board of Education approve the addition of Advanced Placement Precalculus, Mobile Game Programming, Game Programming Technologies, and Front-End Development with Angular to the course catalog for the 2023-24 school year.

Dr. Jeffrey Weiss
Superintendent of Schools

Mrs. Julie Housaman
Chief Academic Officer

Mrs. Jennifer Lawler
Coordinator of Secondary Mathematics and Science

Mr. Aaron Williams
Coordinator of Career and Technical Education

Dr. Bethany Ormseth
Principal, LakeView Technology Academy

APPENDIX A

COURSE CHANGE PROPOSAL

*Completed forms must be returned to the chief academic officer by **October 1** to be considered for board approval.*

Date Initiated: 10/17/22 Administrator's Name: Jennifer Lawler

Department and School: Teaching and Learning

Course Name: Advanced Placement Precalculus

Request: ☒ New Course ☐ New Course Name ☐ Course Revision ☐ Remove Course

Credits: 1.0 *Check if honors:* ☒

Recommended Prerequisites (if any): Algebra 2 Trig, Algebra 2 Modeling & Statistics

Rationale: Explain why this course is needed. (If this is a course removal or name change, only fill out this section.)

Advanced Placement Precalculus will prepare students who start Algebra 1 in ninth grade for a successful transition into a science, technology, engineering, and mathematics (STEM) major in college. These students deserve solid preparation for STEM majors and careers. Providing them with an advanced placement (AP) credit opportunity for precalculus will motivate many students to persist in four years of high school math and will significantly boost student readiness for the subsequent math classes they will need to major in STEM.

Advanced Placement Precalculus will help students who take Algebra 1 before ninth grade prepare for Advanced Placement Calculus. Students who take Advanced Placement Precalculus before their senior year will be much better prepared for success in Advanced Placement Calculus and for any subsequent math they may need in college.

Proposed Course Description: In three or four sentences, write a course overview.

Advanced Placement Precalculus fosters the development of a deep conceptual understanding of functions. In this course students study a broad spectrum of function types that are foundational for careers in mathematics, physics, biology, health science, social science, and data science. Students will apply mathematical tools in real-world modeling situations in preparation for using these tools in college-level calculus as well as the symbolic manipulation skills needed for future mathematics courses. This course prepares students for college-level calculus, including Advanced Placement calculus and provides grounding for other mathematics and science courses.

Content Standards and Benchmarks: List the primary content standards and benchmarks students will be expected to understand and be able to apply as a result of taking this course. (Attach additional documents as needed.)

See attachment.

Scope and Sequence: Outline the planned structure for the course, including a tentative timeline for instruction. (Attach additional documents as needed.)

See attachment.

Cost Associated with the Course: Estimate the costs involved in offering this course. List desired texts and materials on a separate sheet. Also list and explain other needs.

A. Teaching Staff: $0

B. Textbooks/Kits: $150,000

C. Supplementary: $0

D. Facilities/Space: $0

E. Professional Learning: $4,000

APPENDIX A
ATTACHMENT 1

CollegeBoard AP®

AP® Precalculus

PROPOSED COURSE FRAMEWORK

May 2022
Preview

Equity and Access

College Board strongly encourages educators to make equitable access a guiding principle for their AP programs by giving all willing and academically prepared students the opportunity to participate in AP. We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial, and socioeconomic groups that have been traditionally underrepresented. Schools should make every effort to ensure their AP classes reflect the diversity of their student population. College Board also believes that all students should have access to academically challenging coursework before they enroll in AP classes, which can prepare them for AP success. It is only through a commitment to equitable preparation and access that true equity and excellence can be achieved.

Contents

Acknowledgements

College Board would like to acknowledge the following individuals for their assistance with and contributions to the development of this course. All individuals' affiliations were current at the time of contribution.

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Natasha Vasavada, *Executive Director, New Course Development*

About AP

College Board's Advanced Placement® Program (AP®) enables willing and academically prepared students to pursue college-level studies—with the opportunity to earn college credit, advanced placement, or both—while still in high school. Through AP courses in 38 subjects, each culminating in a challenging exam, students learn to think critically, construct solid arguments, and see many sides of an issue—skills that prepare them for college and beyond. Taking AP courses demonstrates to college admission officers that students have sought the most challenging curriculum available to them, and research indicates that students who score a 3 or higher on an AP Exam typically experience greater academic success in college and are more likely to earn a college degree than non-AP students. Each AP teacher's syllabus is evaluated and approved by faculty from some of the nation's leading colleges and universities, and AP Exams are developed and scored by college faculty and experienced AP teachers. Most four-year colleges and universities in the United States grant credit, advanced placement, or both on the basis of successful AP Exam scores; more than 3,300 institutions worldwide annually receive AP scores.

AP Course Development

In an ongoing effort to maintain alignment with best practices in college-level learning, AP courses and exams emphasize challenging, research-based curricula aligned with higher education expectations.

Individual teachers are responsible for designing their own curriculum for AP courses and selecting appropriate college-level readings, assignments, and resources. This course framework document presents the content and skills that are the focus of the corresponding college course and that appear on the AP Exam. It also organizes the content and skills into a series of units that represent a sequence found in widely adopted college textbooks and that many AP teachers have told us they follow in order to focus their instruction. The intention of this publication is to respect teachers' time and expertise by providing a roadmap that they can modify and adapt to their local priorities and preferences. Moreover, by organizing the AP course content and skills into units, the AP Program is able to provide teachers and students with formative assessments—Progress Checks—that teachers can assign throughout the year to measure students' progress as they acquire content knowledge and develop skills.

Enrolling Students: Equity and Access

College Board strongly encourages educators to make equitable access a guiding principle for their AP programs by giving all willing and academically prepared students the opportunity to participate in AP. We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial, and socioeconomic groups that have been traditionally underserved. College Board also believes that all students should have access to academically challenging coursework before they enroll in AP classes, which can prepare them for AP success. It is only through a commitment to equitable preparation and access that true equity and excellence can be achieved.

Offering AP Courses: The AP Course Audit

The AP Program unequivocally supports the principle that each school implements its own curriculum that will enable students to develop the content understandings and skills described in the course framework.

While the unit sequence presented in this publication is optional, the AP Program does have a short list of curricular and resource requirements that must be fulfilled before a school can label a course "Advanced Placement" or "AP." Schools wishing to offer AP courses must participate in the AP Course Audit, a process through which AP teachers' course materials are reviewed by college faculty. The AP Course Audit was created to provide teachers and administrators with clear guidelines on curricular and resource requirements for AP courses and to help colleges and universities validate courses marked "AP" on students' transcripts. This process ensures that AP teachers' courses meet or exceed the curricular and resource expectations that college and secondary school faculty have established for college-level courses.

The AP Course Audit form is submitted by the AP teacher and the school principal (or designated administrator) to confirm awareness and understanding of the curricular and resource requirements. A syllabus or course outline, detailing how course requirements are met, is submitted by the AP teacher for review by college faculty.

Please visit **collegeboard.org/apcourseaudit** for more information to support the preparation and submission of materials for the AP Course Audit.

How the AP Program Is Developed

The scope of content for an AP course and exam is derived from an analysis of hundreds of syllabi and course offerings of colleges and universities. Using this research and data, a committee of college faculty and expert AP teachers work within the scope of the corresponding college course to articulate what students should know and be able to do upon the completion of the AP course. The resulting course framework serves as a blueprint of the content and skills that can appear on an AP Exam.

The AP Test Development Committees are responsible for developing each AP Exam, ensuring the exam questions are aligned to the course framework. The AP Exam development process is a multiyear endeavor; all AP Exams undergo extensive review, revision, and analysis to ensure that questions are accurate, fair, and valid and that there is an appropriate spread of difficulty across the questions.

Committee members are selected to represent a variety of perspectives and institutions (public and private, small and large schools and colleges) and a range of gender, racial/ethnic, and regional groups. A list of each subject's current AP Test Development Committee members is available on **apcentral.collegeboard.org**.

Throughout AP course and exam development, College Board gathers feedback from various stakeholders in both secondary schools and higher education institutions. This feedback is carefully considered to ensure that AP courses and exams are able to provide students with a college-level learning experience and the opportunity to demonstrate their qualifications for advanced placement or college credit.

How AP Exams Are Scored

The exam scoring process, like the course and exam development process, relies on the expertise of both AP teachers and college faculty. While multiple-choice questions are scored by machine, the free-response questions and through-course performance assessments, as applicable, are scored by thousands of college faculty and expert AP teachers. Most are scored at the annual AP Reading, while a small portion are scored online. All AP readers are thoroughly trained, and their work is monitored throughout the Reading for fairness and consistency. In each subject, a highly respected college faculty member serves as Chief Faculty Consultant and, with the help of AP readers in leadership positions, maintains the accuracy of the scoring standards. Scores on the free-response questions and performance assessments are weighted and combined with the results of the computer-scored multiple-choice questions, and this raw score is converted into a composite AP score on a 1–5 scale.

AP Exams are **not** norm-referenced or graded on a curve. Instead, they are criterion-referenced, which means that every student who meets the criteria for an AP score of 2, 3, 4, or 5 will receive that score, no matter how many students that is. The criteria for the number of points students must earn on the AP Exam to receive scores of 3, 4, or 5—the scores that research consistently validates for credit and placement purposes—include:

- The number of points successful college students earn when their professors administer AP Exam questions to them
- The number of points researchers have found to be predictive that an AP student will succeed when placed into a subsequent, higher-level college course
- Achievement-level descriptions formulated by college faculty who review each AP Exam question

AP PRECALCULUS

Course Framework

Introduction

AP Precalculus centers on functions modeling dynamic phenomena. This research-based exploration of functions is designed to better prepare students for college-level calculus and provide grounding for other mathematics and science courses. In this course, students study a broad spectrum of function types that are foundational for careers in mathematics, physics, biology, health science, social science, and data science. Furthermore, as AP Precalculus may be the last mathematics course of a student's secondary education, the course is structured to provide a coherent capstone experience and is not exclusively focused on preparation for future courses.

During this course, students acquire and apply mathematical tools in real-world modeling situations in preparation for using these tools in college-level calculus. Modeling, a central instructional theme for the course, helps students come to a deeper understanding of each function type. By examining scenarios, conditions, and data sets, as well as determining and validating an appropriate function model, students develop a greater comprehension of the nature and behavior of the function itself. The formal study of a function type through multiple representations (e.g., graphical, numerical, verbal, analytical), coupled with the application of the function type to a variety of contexts, provides students with a rich study of precalculus.

Throughout this course, students develop and hone symbolic manipulation skills needed for future mathematics courses. They also solve equations and manipulate expressions for the many function types throughout the course. Students also learn that functions and their compositions, inverses, and transformations are understood through graphical, numerical, verbal, and analytical representations, which reveal different attributes of the functions and are useful for solving problems in mathematical and applied contexts. In turn, the skills learned in this course are widely applicable in a variety of future courses that involve quantitative reasoning.

AP Precalculus fosters the development of a deep conceptual understanding of functions. Students learn that a function is a mathematical relation that maps a set of input values—the domain—to a set of output values—the range—such that each input value is uniquely mapped to an output value. At various points and over various intervals, a function takes on characteristics that can be classified with varying levels of precision and justification, depending on the function representation and available mathematical tools. Furthermore, a function can be classified as part of a function family based on the way in which values of different variables change simultaneously.

Research indicates that deep understanding of functions and their graphs as embodying dynamic covariation of quantities best supports student preparation for calculus. With each function type, students develop and validate function models based on the characteristics of a bivariate data set, characteristics of covarying quantities and their relative rates of change, or a set of characteristics such as zeros, asymptotes, and extrema. These models are used to interpolate, extrapolate, and interpret information with varying degrees of accuracy for a given context or data set. Additionally, students also learn that every model is subject to assumptions and limitations related to the context. As a result of examining functions from many perspectives, students develop a conceptual understanding not only of specific function types but also of functions in general. This type of understanding helps students to engage with both familiar and novel contexts.

Unit Outline

Unit 1: Polynomial and Rational Functions

Unit 2: Exponential and Logarithmic Functions

Unit 3: Trigonometric and Polar Functions

Unit 4: Functions Involving Parameters, Vectors, and Matrices

Unit Notes

Each unit includes these features:

- Exploration, analysis, and application of **new function types**.
- Deep development of a **key function concept** applicable across function types such as transformations, compositions, and inverses.
- Examination of **how variables change relative to each other** for each of the function types.
- Use of each function type to **model contexts and data sets**.
- Rigorous application of the **algebraic skills** needed to engage with each function type.

Technology Notes

Technology should be used throughout the course as a tool to explore concepts. In AP Precalculus, students should specifically practice using technology to do the following:

- Perform calculations (e.g., exponents, roots, trigonometric values, logarithms)
- Graph functions and analyze graphs
- Generate a table of values for a function
- Find real zeros of functions
- Find points of intersection of graphs of functions
- Find minima/maxima of functions
- Find numerical solutions to equations in one variable
- Find regressions equations to model data
- Perform matrix operations (e.g., multiplication, finding inverses)

However, it is important to note that technology should not replace the development of symbolic manipulation skills. When algebraic expressions and equations are accessible with precalculus-level algebraic manipulation, students are expected to find zeros, solve equations, and calculate values without the help of technology. Most of the AP Exam will need to be completed without the use of technology. However, selected questions will require students to use a graphing calculator to complete the tasks delineated above.

Expected Prior Knowledge and Skills

- Proficiency with linear functions
- Proficiency in polynomial addition and multiplication
- Proficiency in factoring quadratic trinomials
- Proficiency in using the quadratic formula
- Proficiency in solving right triangle problems involving trigonometry
- Proficiency in solving linear and quadratic equations and inequalities
- Proficiency in algebraic manipulation of linear equations and expressions
- Proficiency in solving systems of equations in two and three variables
- Familiarity with piecewise-defined functions
- Familiarity with exponential functions and rules for exponents
- Familiarity with radicals (e.g., square roots, cube roots)
- Familiarity with complex numbers

Course Framework Components

Overview

This course framework provides a clear and detailed description of the course requirements necessary for student success. The framework specifies what students should know, be able to do, and understand to qualify for college credit or placement.

The course framework includes two essential components:

MATHEMATICAL PRACTICES

The mathematical practices are central to the study and practice of precalculus. Students should develop and apply the described skills on a regular basis over the span of the course.

COURSE CONTENT

The course content is organized into units of study that provide a suggested sequence for the course. These units comprise the content and conceptual understandings that colleges and universities typically expect students to master to qualify for college credit and/or placement.

COURSE FRAMEWORK CONVENTIONS:

Common language usage (e.g., "area of a triangle") replaces precise mathematical phrasing (e.g., "area of the interior of a triangle") in the following instances:

- When the framework refers to modeling a data set, it is referring to a bivariate data set.
- When the framework refers to modeling a context or phenomenon, it is referring to two aspects of the context or phenomena.
- When the framework refers to the sine, cosine, and so on of an angle, it is referring to the sine, cosine, and so on of the measure of the angle.

Mathematical Practices

The eight distinct skills are associated with three mathematical practices. Students should build and master these skills throughout the course. While many different skills can be applied to any one content topic, the framework supplies skill focus recommendations for each topic to help assure skill distribution throughout the course.

Practice 1	Practice 2	Practice 3
Procedural and Symbolic Fluency Algebraically manipulate functions, equations, and expressions.	**Multiple Representations** Translate mathematical information between representations.	**Communication and Reasoning** Communicate with precise language, and provide rationales for conclusions.
Skill 1.A: Solve equations and inequalities represented analytically, with and without technology. **Skill 1.B: Express** functions, equations, or expressions in analytically **equivalent forms** that are useful in a given mathematical or applied context. **Skill 1.C: Construct new functions**, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.	**Skill 2.A: Identify information from** graphical, numerical, analytical, and verbal **representations** to answer a question or construct a model, with and without technology. **Skill 2.B: Construct equivalent** graphical, numerical, analytical, and verbal **representations** of functions that are useful in a given mathematical or applied context, with and without technology.	**Skill 3.A: Describe** the **characteristics** of a function with varying levels of precision, depending on the function representation and available mathematical tools. **Skill 3.B: Apply** numerical **results** in a given mathematical or applied context. **Skill 3.C: Support conclusions** or choices with a logical rationale or appropriate data.

Course at a Glance

Unit 1	Polynomial and Rational Functions 6–6.5 weeks
1.1	Change in Tandem
1.2	Rates of Change
1.3	Rates of Change in Linear and Quadratic Functions
1.4	Polynomial Functions and Rates of Change
1.5	Polynomial Functions and Complex Zeros
1.6	Polynomial Functions and End Behavior
1.7	Rational Functions and End Behavior
1.8	Rational Functions and Zeros
1.9	Rational Functions and Vertical Asymptotes
1.10	Rational Functions and Holes
1.11	Equivalent Representations of Polynomial and Rational Expressions
1.12	Transformations of Functions
1.13	Function Model Selection and Assumption Articulation
1.14	Function Model Construction and Application

Unit 2	Exponential and Logarithmic Functions 6–6.5 weeks
2.1	Change in Arithmetic and Geometric Sequences
2.2	Change in Linear and Exponential Functions
2.3	Exponential Functions
2.4	Exponential Function Manipulation
2.5	Exponential Function Context and Data Modeling
2.6	Competing Function Model Validation
2.7	Composition of Functions
2.8	Inverse Functions
2.9	Logarithmic Expressions
2.10	Inverses of Exponential Functions
2.11	Logarithmic Functions
2.12	Logarithmic Function Manipulation
2.13	Exponential and Logarithmic Equations and Inequalities
2.14	Logarithmic Function Context and Data Modeling
2.15	Semi-log Plots

Unit 3	Trigonometric and Polar Functions 7–7.5 weeks
3.1	Periodic Phenomena
3.2	Sine, Cosine, and Tangent
3.3	Sine and Cosine Function Values
3.4	Sine and Cosine Function Graphs
3.5	Sinusoidal Functions
3.6	Sinusoidal Function Transformations
3.7	Sinusoidal Function Context and Data Modeling
3.8	The Tangent Function
3.9	Inverse Trigonometric Functions
3.10	Trigonometric Equations and Inequalities
3.11	The Secant, Cosecant, and Cotangent Functions
3.12	Equivalent Representations of Trigonometric Functions
3.13	Trigonometry and Polar Coordinates
3.14	Polar Function Graphs
3.15	Rates of Change in Polar Functions

Unit 4	Functions Involving Parameters, Vectors, and Matrices 7–7.5 weeks
4.1	Parametric Functions
4.2	Parametric Functions Modeling Planar Motion
4.3	Parametric Functions and Rates of Change
4.4	Parametrically Defined Circles and Lines
4.5	Implicitly Defined Functions
4.6	Conic Sections
4.7	Parametrization of Implicitly Defined Functions
4.8	Vectors
4.9	Vector-Valued Functions
4.10	Matrices
4.11	The Inverse and Determinant of a Matrix
4.12	Linear Transformations and Matrices
4.13	Matrices as Functions
4.14	Matrices Modeling Contexts

UNIT 1:

Polynomial and Rational Functions

6–6.5 WEEKS

Unit at a Glance

Topic #	Topic Title	Instructional Periods	Suggested Skill Focus
1.1	Change in Tandem	2	2.B, 3.A
1.2	Rates of Change	2	2.A, 3.A
1.3	Rates of Change in Linear and Quadratic Functions	2	3.B, 3.C
1.4	Polynomial Functions and Rates of Change	2	2.A, 3.A
1.5	Polynomial Functions and Complex Zeros	2	1.B, 2.B
1.6	Polynomial Functions and End Behavior	1	3.A
1.7	Rational Functions and End Behavior	2	1.B, 3.A
1.8	Rational Functions and Zeros	1	1.A
1.9	Rational Functions and Vertical Asymptotes	1	2.A
1.10	Rational Functions and Holes	1	3.C
1.11	Equivalent Representations of Polynomial and Rational Expressions	2	1.B, 3.B
1.12	Transformations of Functions	2	1.C, 3.A
1.13	Function Model Selection and Assumption Articulation	2	2.A, 3.C
1.14	Function Model Construction and Application	2	1.C, 3.B

TOPIC 1.1

Change in Tandem

Instructional Periods: 2
Skills Focus: 2.B, 3.A

LEARNING OBJECTIVES

1.1.A Describe how the input and output values of a function vary together by comparing function values.

1.1.B Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.

ESSENTIAL KNOWLEDGE

- **1.1.A.1** A *function* is a mathematical relation that maps a set of input values to a set of output values such that each input value is mapped to exactly one output value. The set of input values is called the *domain* of the function, and the set of output values is called the *range* of the function. The variable representing input values is called the *independent variable*, and the variable representing output values is called the *dependent variable*.
- **1.1.A.2** The input and output values of a function vary in tandem according to the *function rule*, which can be expressed graphically, numerically, analytically, or verbally.
- **1.1.A.3** A function is increasing over an interval of its domain if, as the input values increase, the output values always increase. That is, for all a and b in the interval, if $a<b$, then $f(a)<f(b)$.
- **1.1.A.4** A function is decreasing over an interval of its domain if, as the input values increase, the output values always decrease. That is, for all a and b in the interval, if $a<b$, then $f(a)>f(b)$.

- **1.1.B.1** The graph of a function displays a set of input-output pairs and shows how the values of the function's input and output values vary.
- **1.1.B.2** A verbal description of the way aspects of phenomena change together can be the basis for constructing a graph.
- **1.1.B.3** The graph of a function is *concave up* on intervals in which the rate of change is increasing.
- **1.1.B.4** The graph of a function is *concave down* on intervals in which the rate of change is decreasing.
- **1.1.B.5** The graph intersects the *x*-axis when the output value is zero. The corresponding input values are said to be *zeros of the function*.

TOPIC 1.2

Rates of Change

Instructional Periods: 2
Skills Focus: 2.A, 3.A

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
1.2.A Compare the rates of change at two points using average rates of change near the points.	▪ **1.2.A.1** The average rate of change of a function over an interval of the function's domain is the constant rate of change that yields the same change in the output values as the function yielded on that interval of the function's domain. It is the ratio of the change in the output values to the change in input values over that interval. ▪ **1.2.A.2** The rate of change of a function at a point quantifies the rate at which output values would change were the input values to change at that point. The rate of change at a point can be approximated by the average rates of change of the function over small intervals containing the point, if such values exist. ▪ **1.2.A.3** The rates of change at two points can be compared using average rate of change approximations over sufficiently small intervals containing each point, if such values exist.
1.2.B Describe how two quantities vary together at different points and over different intervals of a function.	▪ **1.2.B.1** Rates of change quantify how two quantities vary together. ▪ **1.2.B.2** A positive rate of change indicates that as one quantity increases or decreases, the other quantity does the same. ▪ **1.2.B.3** A negative rate of change indicates that as one quantity increases, the other decreases.

TOPIC 1.3

Rates of Change in Linear and Quadratic Functions

Instructional Periods: 2
Skills Focus: 3.B, 3.C

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
1.3.A Determine the average rates of change for linear and quadratic sequences and functions.	▪ **1.3.A.1** Over any length input-value interval, the average rate of change for a linear function is constant. ▪ **1.3.A.2** For consecutive equal-length input-value intervals, the average rate of change of a quadratic function can be given by a linear function. ▪ **1.3.A.3** The average rate of change over the closed interval $[a, b]$ is the slope of the secant line from the point $(a, f(a))$ to $(b, f(b))$.
1.3.B Determine the change of average rates of change for linear and quadratic functions.	▪ **1.3.B.1** Because the average rate of change of a linear function over any length input-value interval is constant, the rate of change of the average rates of change of a linear function is zero. ▪ **1.3.B.2** Because the average rate of change of a quadratic function over consecutive equal-length input-value intervals can be given by a linear function, the rate of change of the average rates of change of a quadratic function is constant. ▪ **1.3.B.3** When the average rate of change over equal-length input-value intervals is increasing for all small-length intervals, the graph of the function is concave up. When the average rate of change over equal-length input-value intervals is decreasing for all small-length intervals, the graph of the function is concave down.

TOPIC 1.4

Polynomial Functions and Rates of Change

Instructional Periods: 2
Skills Focus: 2.A, 3.A

LEARNING OBJECTIVES

1.4.A Identify key characteristics of polynomial functions related to rates of change.

ESSENTIAL KNOWLEDGE

- **1.4.A.1** A nonconstant polynomial function of x is any function representation that is equivalent to the analytical form $p(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots+a_2x^2+a_1x+a_0$, where n is a positive integer, a_i is a real number for each i from 1 to n, and a_n is nonzero. The polynomial has degree n, the leading term is a_nx^n, and the leading coefficient is a_n. A constant is also a polynomial function of degree zero.
- **1.4.A.2** Where a polynomial function switches between increasing and decreasing, or at the included endpoint of a polynomial with a restricted domain, the polynomial function will have a *local*, or *relative*, maximum or minimum output value. Of all local maxima, the greatest is called the *global*, or *absolute*, maximum. Likewise, the least of all local minima is called the global, or absolute, minimum.
- **1.4.A.3** Between every two distinct real zeros of a nonconstant polynomial function, there must be at least one input value corresponding to a local maximum or local minimum.
- **1.4.A.4** Polynomial functions of an even degree will have either a global maximum or a global minimum.
- **1.4.A.5** *Points of inflection* of a polynomial function occur at input values where the rate of change of the function changes from increasing to decreasing or from decreasing to increasing. This occurs where the graph of a polynomial function changes from concave up to concave down or from concave down to concave up.

TOPIC 1.5

Polynomial Functions and Complex Zeros

Instructional Periods: 2
Skills Focus: 1.B, 2.B

LEARNING OBJECTIVES

1.5.A Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology.

ESSENTIAL KNOWLEDGE

- **1.5.A.1** If a is a complex number and $p(a)=0$, then a is called a *zero* of p, or a *root* of the polynomial function p. If a is a real number, then $(x-a)$ is a linear factor of p if and only if a is a zero of p.
- **1.5.A.2** If a linear factor $(x-a)$ is repeated n times, the corresponding zero of the polynomial function has a *multiplicity* n. A polynomial of *degree* n has exactly n complex zeros when counting multiplicities.
- **1.5.A.3** If a is a real root of a polynomial function p, then the graph of $y=p(x)$ has an *x*-intercept at the point $(a,0)$. Consequently, real zeros of a polynomial can be endpoints for intervals satisfying polynomial inequalities.
- **1.5.A.4** If $a+bi$ is a non-real zero of a polynomial p, then its *conjugate* $a-bi$ is also a zero of p.
- **1.5.A.5** If the real zero, a, of a polynomial function has even multiplicity, then the signs of the output values are the same for input values near $x=a$. For these polynomials, the graph will be tangent to the *x*-axis at $x=a$.
- **1.5.A.6** The degree of a polynomial function can be found by examining the successive differences of the output values over equal-interval input values. The degree of the polynomial function is equal to the least value n for which the successive nth differences are constant.

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
1.5.B Determine if a polynomial is even or odd.	▪ **1.5.B.1** An *even* function is graphically symmetric over the line $x = 0$ and analytically has the property $f(-x) = f(x)$. If n is even, then a polynomial of the form $p(x) = a_n x^n$, where $n \geq 1$ and $a_n \neq 0$, is an even function. ▪ **1.5.B.2** An *odd* function is graphically symmetric about the point $(0, 0)$ and analytically has the property $f(-x) = -f(x)$. If n is odd, then a polynomial of the form $p(x) = a_n x^n$, where $n \geq 1$ and $a_n \neq 0$, is an odd function.

TOPIC 1.6

Polynomial Functions and End Behavior

Instructional Periods: 1
Skills Focus: 3.A

LEARNING OBJECTIVES

1.6.A Describe end behaviors of polynomial functions.

ESSENTIAL KNOWLEDGE

- **1.6.A.1** As input values of a nonconstant polynomial function increase without bound, the output values will either increase or decrease without bound. The corresponding mathematical notation is $\lim_{x\to\infty} p(x)=\infty$ or $\lim_{x\to\infty} p(x)=-\infty$.
- **1.6.A.2** As input values of a nonconstant polynomial function decrease without bound, the output values will either increase or decrease without bound. The corresponding mathematical notation is $\lim_{x\to-\infty} p(x)=\infty$ or $\lim_{x\to-\infty} p(x)=-\infty$.
- **1.6.A.3** The degree and sign of the leading term of a polynomial determines the end behavior of the polynomial function, because as the input values increase or decrease without bound, the values of the leading term dominate the values of all lower-degree terms.

TOPIC 1.7

Rational Functions and End Behavior

Instructional Periods: 2
Skills Focus: 1.B, 3.A

LEARNING OBJECTIVES

1.7.A Describe end behaviors of rational functions.

ESSENTIAL KNOWLEDGE

- **1.7.A.1** A rational function is analytically represented as a quotient of two polynomial functions and gives a measure of the relative size of the polynomial function in the numerator compared to the polynomial function in the denominator for each value in the rational function's domain.
- **1.7.A.2** The end behavior of a rational function will be affected most by the polynomial with the greater degree, as its values will dominate the values of the rational function for input values of large magnitude. For input values of large magnitude, a polynomial is dominated by its leading term. Therefore, the end behavior of a rational function can be understood by examining the corresponding quotient of the leading terms.
- **1.7.A.3** If the polynomial in the numerator dominates the polynomial in the denominator for input values of large magnitude, then the quotient of the leading terms is a nonconstant polynomial, and the original rational function has the end behavior of that polynomial. If that polynomial is linear, then the rational function has a slant asymptote parallel to the graph of the line.
- **1.7.A.4** If neither polynomial in a rational function dominates the other for input values of large magnitude, then the quotient of the leading terms is a constant, and that constant indicates the location of a horizontal asymptote of the original rational function.
- **1.7.A.5** If the polynomial in the denominator dominates the polynomial in the numerator for input values of large magnitude, then the quotient of the leading terms is a rational function with a constant in the numerator and nonconstant polynomial in the denominator, and the original rational function has a horizontal asymptote at $y=0$.
- **1.7.A.6** When the graph of a rational function has a horizontal asymptote $y=b$, where b is a constant, the output values of the rational function get arbitrarily close to b and stay arbitrarily close to b as input values increase or decrease without bound. The corresponding mathematical notation is $\lim_{x\to\infty} r(x)=b$ or $\lim_{x\to-\infty} r(x)=b$.

TOPIC 1.8

Rational Functions and Zeros

Instructional Periods: 1
Skills Focus: 1.A

LEARNING OBJECTIVES

1.8.A Determine the zeros of rational functions.

ESSENTIAL KNOWLEDGE

- **1.8.A.1** The real zeros of a rational function correspond to the real zeros of the numerator for such values in its domain.
- **1.8.A.2** The real zeros of both polynomials of a rational function are endpoints or asymptotes for intervals satisfying the rational function inequalities $r(x) \geq 0$ or $r(x) \leq 0$.

TOPIC 1.9

Rational Functions and Vertical Asymptotes

Instructional Periods: 1
Skills Focus: 2.A

LEARNING OBJECTIVES

1.9.A Determine vertical asymptotes of rational functions.

ESSENTIAL KNOWLEDGE

- **1.9.A.1** If the value a is a real zero of the polynomial in the denominator of a rational function and is not also a real zero of the polynomial in the numerator, then the graph of the rational function has a vertical asymptote at $x = a$. Furthermore, a vertical asymptote also occurs at $x = a$ if the multiplicity of a as a real zero in the denominator is greater than its multiplicity as a real zero in the numerator.
- **1.9.A.2** Near a vertical asymptote, $x = a$, of a rational function, the values of the polynomial in the denominator are arbitrarily close to zero, so the values of the rational function increase or decrease without bound. The corresponding mathematical notation is $\lim_{x \to a^+} r(x) = \infty$ or $\lim_{x \to a^+} r(x) = -\infty$ for input values near a and greater than a, and $\lim_{x \to a^-} r(x) = \infty$ or $\lim_{x \to a^-} r(x) = -\infty$ for input values near a and less than a.

TOPIC 1.10

Rational Functions and Holes

Instructional Periods: 1
Skills Focus: 3.C

LEARNING OBJECTIVES

1.10.A Determine holes in graphs of rational functions.

ESSENTIAL KNOWLEDGE

- **1.10.A.1** If the multiplicity of a real zero in the numerator is greater than or equal to its multiplicity in the denominator, then the graph of the rational function has a hole at the corresponding input value.
- **1.10.A.2** If the graph of a rational function has a hole at $x = c$, then the location of the hole can be determined by examining the output values corresponding to input values sufficiently close to c. If input values sufficiently close to c correspond to output values arbitrarily close to L, then the hole is located at the point with coordinates (c, L). The corresponding mathematical notation is $\lim_{x \to c} r(x) = L$. It should be noted that $\lim_{x \to c^-} r(x) = \lim_{x \to c^+} r(x) = \lim_{x \to c} r(x) = L$.

TOPIC 1.11

Equivalent Representations of Polynomial and Rational Expressions

Instructional Periods: 2
Skills Focus: 1.B, 3.B

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
1.11.A Rewrite polynomial and rational expressions in equivalent forms.	▪ **1.11.A.1** Because the factored form of a polynomial or rational function readily provides information about real zeros, it can reveal information about *x*-intercepts, asymptotes, holes, domain, and range. ▪ **1.11.A.2** The standard form of a polynomial or rational function can reveal information about end behavior of the function. ▪ **1.11.A.3** The information extracted from different analytic representations of the same polynomial or rational function can be used to answer questions in context.
1.11.B Determine the quotient of two polynomials using long division.	▪ **1.11.B.1** Polynomial long division is an algebraic process similar to numerical long division involving a quotient and remainder. If the polynomial f is divided by the polynomial g, then f can be rewritten as $f(x)=g(x)q(x)+r(x)$, where q is the quotient, r is the remainder and the degree of r is less than the degree of g. ▪ **1.11.B.2** The result of polynomial long division is helpful in finding equations of slant asymptotes.
1.11.C Rewrite the repeated product of binomials using the binomial theorem.	▪ **1.11.C.1** The binomial theorem utilizes the entries in a single row of Pascal's Triangle to more easily expand expressions of the form $(a+b)^n$, including polynomial functions of the form $p(x)=(x+c)^n$, where c is a constant.

TOPIC 1.12

Transformations of Functions

Instructional Periods: 2
Skills Focus: 1.C, 3.A

LEARNING OBJECTIVES

1.12.A Construct a function that is an additive and/or multiplicative transformation of another function.

ESSENTIAL KNOWLEDGE

- **1.12.A.1** The function $g(x)=f(x)+k$ is an additive transformation of the function f that results in a vertical translation of the graph of f by k units.
- **1.12.A.2** The function $g(x)=f(x+h)$ is an additive transformation of the function f that results in a horizontal translation of the graph of f by $-h$ units.
- **1.12.A.3** The function $g(x)=af(x)$, where $a\neq 0$, is a multiplicative transformation of the function f that results in a vertical dilation of the graph of f by a factor of $|a|$. If $a<0$, the transformation involves a reflection over the *x*-axis.
- **1.12.A.4** The function $g(x)=f(bx)$, where $b\neq 0$, is a multiplicative transformation of the function f that results in a horizontal dilation of the graph of f by a factor of $\left|\frac{1}{b}\right|$. If $b<0$, the transformation involves a reflection over the *y*-axis.
- **1.12.A.5** Additive and multiplicative transformations can be combined, resulting in combinations of horizontal and vertical translations and dilations.
- **1.12.A.6** The domain and range of a function that is a transformation of a parent function may be different from those of the parent function.

TOPIC 1.13

Function Model Selection and Assumption Articulation

Instructional Periods: 2
Skills Focus: 2.A, 3.C

LEARNING OBJECTIVES

1.13.A Identify an appropriate function type to construct a function model for a given scenario.

ESSENTIAL KNOWLEDGE

- **1.13.A.1** Linear functions model data sets or aspects of contextual scenarios that demonstrate roughly constant rates of change.
- **1.13.A.2** Quadratic functions model data sets or aspects of contextual scenarios that demonstrate roughly linear rates of change, or data sets that are roughly symmetric with a unique maximum or minimum value.
- **1.13.A.3** Geometric contexts involving area or two dimensions can often be modeled by quadratic functions. Geometric contexts involving volume or three dimensions can often be modeled by cubic functions.
- **1.13.A.4** Polynomial functions model data sets or contextual scenarios with multiple real zeros or multiple maxima or minima.
- **1.13.A.5** A polynomial function of degree n models data sets or contextual scenarios that demonstrate roughly constant nonzero nth differences.
- **1.13.A.6** A polynomial function of degree n or less can be used to model a graph of $n+1$ points with distinct input values.
- **1.13.A.7** A piecewise-defined function consists of a set of functions defined over nonoverlapping domain intervals and is useful for modeling a data set or contextual scenario that demonstrates different characteristics over different intervals.

LEARNING OBJECTIVES

1.13.B Describe assumptions and restrictions related to building a function model.

ESSENTIAL KNOWLEDGE

- **1.13.B.1** A model may have underlying assumptions about what is consistent in the model.
- **1.13.B.2** A model may have underlying assumptions about how quantities change together.
- **1.13.B.3** A model may require domain restrictions based on mathematical clues, contextual clues, or extreme values in the data set.
- **1.13.B.4** A model may require range restrictions, such as rounding values, based on mathematical clues, contextual clues, or extreme values in the data set.

TOPIC 1.14

Function Model Construction and Application

Instructional Periods: 2
Skills Focus: 1.C, 3.B

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
1.14.A Construct a linear, polynomial, or related piecewise-defined function model.	▪ **1.14.A.1** A model can be constructed based on restrictions identified in a mathematical or contextual scenario. ▪ **1.14.A.2** A model of a data set or a contextual scenario can be constructed using transformations of the parent function. ▪ **1.14.A.3** A model of a data set can be constructed using technology and regressions. ▪ **1.14.A.4** A piecewise-defined function model can be constructed through a combination of modeling techniques.
1.14.B Construct a rational function model based on a context.	▪ **1.14.B.1** Data sets and aspects of contextual scenarios involving quantities that are inversely proportional can often be modeled by rational functions. For example, the magnitudes of both gravitational force and electromagnetic force between objects are inversely proportional to the objects' squared distance.
1.14.C Apply a function model to answer questions about a data set or contextual scenario.	▪ **1.14.C.1** A model can be used to draw conclusions about the modeled data set or contextual scenario, including answering key questions and predicting values, rates of change, average rates of change, and changing rates of change. Appropriate units of measure should be extracted or inferred from the given context.

UNIT 2:

Exponential and Logarithmic Functions

6–6.5 WEEKS

Unit at a Glance

Topic #	Topic Title	Instructional Periods	Suggested Skill Focus
2.1	Change in Arithmetic and Geometric Sequences	2	1.B, 3.A
2.2	Change in Linear and Exponential Functions	2	1.C, 3.B
2.3	Exponential Functions	1	3.A
2.4	Exponential Function Manipulation	2	1.B, 3.A
2.5	Exponential Function Context and Data Modeling	2	1.C, 3.B
2.6	Competing Function Model Validation	2	2.A, 3.C
2.7	Composition of Functions	2	1.C, 2.B
2.8	Inverse Functions	2	1.A, 2.B
2.9	Logarithmic Expressions	1	1.B
2.10	Inverses of Exponential Functions	2	1.C, 2.B
2.11	Logarithmic Functions	1	3.A
2.12	Logarithmic Function Manipulation	2	1.B, 3.A
2.13	Exponential and Logarithmic Equations and Inequalities	3	1.A, 1.B, 1.C
2.14	Logarithmic Function Context and Data Modeling	2	1.C, 3.B
2.15	Semi-log Plots	2	2.B, 3.C

TOPIC 2.1

Change in Arithmetic and Geometric Sequences

Instructional Periods: 2
Skills Focus: 1.B, 3.A

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
2.1.A Express arithmetic sequences found in mathematical and contextual scenarios as functions of the natural numbers.	▪ **2.1.A.1** A *sequence* is a function from the whole numbers to the real numbers. Consequently, the graph of a sequence consists of discrete points instead of a curve. ▪ **2.1.A.2** Successive terms in an arithmetic sequence have a common difference, or constant rate of change. ▪ **2.1.A.3** The nth term of an arithmetic sequence with a common difference d is denoted by a_n and is given by $a_n = a_0 + dn$, where a_0 is the initial value, or by $a_n = a_k + d(n-k)$, where a_k is the kth term of the sequence.
2.1.B Express geometric sequences found in mathematical and contextual scenarios as functions of the natural numbers.	▪ **2.1.B.1** Successive terms in a geometric sequence have a common ratio, or constant proportional change. ▪ **2.1.B.2** The nth term of a geometric sequence with a common ratio r is denoted by g_n and is given by $g_n = g_0 r^n$, where g_0 is the initial value, or by $g_n = g_k r^{(n-k)}$, where g_k is the kth term of the sequence. ▪ **2.1.B.3** Increasing arithmetic sequences increase equally with each step, whereas increasing geometric sequences increase by a larger amount with each successive step.

TOPIC 2.2

Change in Linear and Exponential Functions

Instructional Periods: 2
Skills Focus: 1.C, 3.B

LEARNING OBJECTIVES

2.2.A Construct functions of the real numbers that are comparable to arithmetic and geometric sequences.

ESSENTIAL KNOWLEDGE

- **2.2.A.1** Linear functions of the form $f(x)=b+mx$ are similar to arithmetic sequences of the form $a_n=a_0+dn$, as both can be expressed as an initial value (b or a_0) plus repeated addition of a constant rate of change, the slope (m or d).
- **2.2.A.2** Similar to arithmetic sequences of the form $a_n=a_k+d(n-k)$, which are based on a known difference, d, and a kth term, linear functions can be expressed in the form $f(x)=y_i+m(x-x_i)$ based on a known slope, m, and a point, (x_i, y_i).
- **2.2.A.3** Exponential functions of the form $f(x)=ab^x$ are similar to geometric sequences of the form $g_n=g_0r^n$, as both can be expressed as an initial value (a or g_0) times repeated multiplication by a constant proportion (b or r).
- **2.2.A.4** Similar to geometric sequences of the form $g_n=g_kr^{(n-k)}$, which are based on a known ratio, r, and a kth term, exponential functions can be expressed in the form $f(x)=y_ir^{(x-x_i)}$ based on a known ratio, r, and a point, (x_i, y_i).
- **2.2.A.5** Sequences and their corresponding functions may have different domains.

LEARNING OBJECTIVES

2.2.B Describe similarities and differences between linear and exponential functions.

ESSENTIAL KNOWLEDGE

- **2.2.B.1** Over equal-length input-value intervals, if the output values of a function change at constant rate, then the function is linear; if the output values of a function change proportionally, then the function is exponential.
- **2.2.B.2** Linear functions of the form $f(x) = b + mx$ and exponential functions of the form $f(x) = ab^x$ can both be expressed analytically in terms of an initial value and a constant involved with change. However, linear functions are based on addition, while exponential functions are based on multiplication.
- **2.2.B.3** Arithmetic sequences, linear functions, geometric sequences, and exponential functions all have the property that they can be determined by two distinct sequence or function values.

TOPIC 2.3

Exponential Functions

Instructional Periods: 1
Skills Focus: 3.A

LEARNING OBJECTIVES

2.3.A Identify key characteristics of exponential functions.

ESSENTIAL KNOWLEDGE

- **2.3.A.1** The general form of an exponential function is $f(x)=ab^x$, with the *initial value* a, where $a \neq 0$, and the *base* b, where $b > 0$, and $b \neq 1$. When $a > 0$ and $b > 1$, the exponential function is said to demonstrate *exponential growth*. When $a > 0$ and $0 < b < 1$, the exponential function is said to demonstrate *exponential decay*.
- **2.3.A.2** When the natural numbers are input values in an exponential function, the input value specifies the number of factors of the base to be applied to the function's initial value. The domain of an exponential function is all real numbers.
- **2.3.A.3** Because the output values of exponential functions in general form are proportional over equal-length input-value intervals, exponential functions are always increasing or always decreasing, and their graphs are always concave up or always concave down. Consequently, exponential functions do not have extrema except on a closed interval, and their graphs do not have inflection points.
- **2.3.A.4** If the values of the additive transformation function $g(x)=f(x)+k$ of any function f are proportional over equal-length input-value intervals, then f is exponential.
- **2.3.A.5** For an exponential function in general form, as the input values increase or decrease without bound, the output values will increase or decrease without bound or will get arbitrarily close to zero. That is, for an exponential function in general form, $\lim_{x \to \pm\infty} ab^x = \infty$, $\lim_{x \to \pm\infty} ab^x = -\infty$, or $\lim_{x \to \pm\infty} ab^x = 0$.

TOPIC 2.4

Exponential Function Manipulation

Instructional Periods: 2
Skills Focus: 1.B, 3.A

LEARNING OBJECTIVES

2.4.A Rewrite exponential expressions in equivalent forms.

ESSENTIAL KNOWLEDGE

- **2.4.A.1** The product property for exponents states that $b^m b^n = b^{(m+n)}$. Graphically, this property implies that every horizontal translation of an exponential function, $f(x) = b^{(x+k)}$, is equivalent to a vertical dilation, $f(x) = b^{(x+k)} = b^x b^k = ab^x$, where $a = b^k$.
- **2.4.A.2** The power property for exponents states that $\left(b^m\right)^n = b^{(mn)}$. Graphically, this property implies that every horizontal dilation of an exponential function, $f(x) = b^{(cx)}$, is equivalent to a change of the base of an exponential function, $f(x) = \left(b^c\right)^x$, where b^c is a constant and $c \neq 0$.
- **2.4.A.3** The negative exponent property states that $b^{-n} = \frac{1}{b^n}$.
- **2.4.A.4** The value of an exponential expression involving an exponential unit fraction, such as $b^{(1/k)}$ where k is a natural number, is the kth root of b, when it exists.

TOPIC 2.5

Exponential Function Context and Data Modeling

Instructional Periods: 2
Skills Focus: 1.C, 3.B

LEARNING OBJECTIVES

2.5.A Construct a model for situations involving proportional output values over equal-length input-value intervals.

ESSENTIAL KNOWLEDGE

- **2.5.A.1** Exponential functions model growth patterns where successive output values over equal-length input-value intervals are proportional. When the input values are whole numbers, exponential functions model situations of repeated multiplication of a constant to an initial value.
- **2.5.A.2** A constant may need to be added to the dependent variable values of a data set to reveal a proportional growth pattern.
- **2.5.A.3** An exponential function model can be constructed from an appropriate ratio and initial value or from two input-output pairs. The initial value and the base can be found by solving a system of equations resulting from the two input-output pairs.
- **2.5.A.4** Exponential function models can be constructed by applying transformations to $f(x)=ab^x$ based on characteristics of a contextual scenario or data set.
- **2.5.A.5** Exponential function models can be constructed for a data set with technology using exponential regressions.
- **2.5.A.6** The natural base e, which is approximately 2.718, is often used as the base in exponential functions that model contextual scenarios.

LEARNING OBJECTIVES

2.5.B Apply exponential models to answer questions about a data set or contextual scenario.

ESSENTIAL KNOWLEDGE

- **2.5.B.1** For an exponential model in general form $f(x) = ab^x$, the base of the exponent, b, can be understood as a growth factor in successive unit changes in the input values and is related to a percent change in context.
- **2.5.B.2** Equivalent forms of an exponential function can reveal different properties of the function. For example, if d represents number of days, then the base of $f(d) = 2^d$ indicates that the quantity increases by a factor of 2 every day, but the equivalent form $f(d) = (2^7)^{(d/7)}$ indicates that the quantity increases by a factor of 2^7 every week.
- **2.5.B.3** Exponential models can be used to predict values for the dependent variable, depending on the contextual constraints on the domain.

TOPIC 2.6

Competing Function Model Validation

Instructional Periods: 2
Skills Focus: 2.A, 3.C

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
2.6.A Construct linear, quadratic, and exponential models based on a data set.	▪ **2.6.A.1** Two variables in a data set that demonstrate a slightly changing rate of change can be modeled by linear, exponential, and quadratic function models. ▪ **2.6.A.2** Models can be compared based on contextual clues and applicability to determine which model is most appropriate.
2.6.B Validate a model constructed from a data set.	▪ **2.6.B.1** A model is justified as *appropriate* for a data set if the graph of the residuals of a regression appear without pattern. ▪ **2.6.B.2** The difference between the predicted and actual values is the *error* in the model. Depending on the data set and context, it may be more appropriate to have an underestimate or overestimate for any given interval.

TOPIC 2.7

Composition of Functions

Instructional Periods: 2
Skills Focus: 1.C, 2.B

LEARNING OBJECTIVES

2.7.A Evaluate the composition of two or more functions for given values.

ESSENTIAL KNOWLEDGE

- **2.7.A.1** If f and g are functions, the composite function $f(g(x))$ maps a set of input values to a set of output values such that the output values of g are used as input values of f. For this reason, the domain of the composite function is restricted to those input values of g for which the corresponding output value is in the domain of f. The composite function $f(g(x))$ uniquely maps input values of g to output values of f, dependent on the domain restrictions of f and g. The composite function $f(g(x))$ can also be represented as $f \circ g(x)$.
- **2.7.A.2** Values for the composite function $f(g(x))$ can be calculated or estimated from the analytical, graphical, numerical, or verbal representations of f and g by using output values from g as input values for f.
- **2.7.A.3** The composition of functions is not commutative; that is, $f(g(x))$ and $g(f(x))$ are typically different functions.
- **2.7.A.4** If the function $f(x)=x$ is composed with any function g, the resulting composite function is the same as g; that is, $g(f(x))=f(g(x))=g(x)$. The function $f(x)=x$ is called the *identity function*. When composing two functions, the identify function acts in the same way as 0, the additive identity, when adding two numbers and 1, the multiplicative identity, when multiplying two numbers.

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
2.7.B Construct a representation of the composition of two or more functions.	▪ **2.7.B.1** Function composition is useful for relating two quantities that are not directly related by an existing formula. ▪ **2.7.B.2** When analytic representations of the functions f and g are available, an analytic representation of $f(g(x))$ can be constructed by substituting $g(x)$ for every instance of x in f. ▪ **2.7.B.3** A numerical or graphical representation of $f(g(x))$ can often be constructed by calculating or estimating values for $(x, f(g(x)))$.
2.7.C Rewrite a given function as a composite of two or more functions.	▪ **2.7.C.1** Functions given analytically can often be decomposed into less complex functions. When properly decomposed, the variable in one function should replace each instance of the function with which it was composed. ▪ **2.7.C.2** An additive transformation of a function, f, that results in vertical and horizontal translations can be understood as the composition of $g(x) = x + k$ with f. ▪ **2.7.C.3** A multiplicative transformation of a function, f, that results in vertical and horizontal dilations can be understood as the composition of $g(x) = kx$ with f.

TOPIC 2.8

Inverse Functions

Instructional Periods: 2
Skills Focus: 1.A, 2.B

LEARNING OBJECTIVES

2.8.A Determine the input-output pairs of the inverse of a function.

ESSENTIAL KNOWLEDGE

- **2.8.A.1** On a specified domain, a function, f, has an inverse function, or is invertible, if each output value of f is mapped from a unique input value. The domain of a function may be restricted in many ways to make the function invertible.
- **2.8.A.2** An inverse function can be thought of as a reverse mapping of the function. An inverse function, f^{-1}, maps the output values of a function, f, on its invertible domain to their corresponding input values; that is, if $f(a)=b$, then $f^{-1}(b)=a$. Alternately, on its invertible domain, if a function consists of input-output pairs (a,b), then the inverse function consists of input-output pairs (b,a).

LEARNING OBJECTIVES

2.8.B Determine the inverse of a function on an invertible domain.

ESSENTIAL KNOWLEDGE

- **2.8.B.1** The composition of a function, f, and its inverse function, f^{-1}, is the identity function; that is, $f\left(f^{-1}(x)\right)=f^{-1}\left(f(x)\right)=x$.
- **2.8.B.2** On a function's invertible domain, the function's range and domain are the inverse function's domain and range, respectively. The inverse of the table of values of $y=f(x)$ can be found by reversing the input-output pairs; that is, (a,b) corresponds to (b,a).
- **2.8.B.3** The inverse of the graph of the function $y=f(x)$ can be found by reversing the roles of the x- and y-axes; that is, by reflecting the graph of the function over the graph of the identity function $h(x)=x$.
- **2.8.B.4** The inverse of the function can be found by determining the inverse operations to reverse the mapping. One method for finding the inverse of the function f is reversing the roles of x and y in the equation $y=f(x)$, then solving for $y=f^{-1}(x)$.
- **2.8.B.5** In addition to limiting the domain of a function to obtain an inverse function, contextual restrictions may also limit the applicability of an inverse function.

TOPIC 2.9

Logarithmic Expressions

Instructional Periods: 1
Skills Focus: 1.B

LEARNING OBJECTIVES

2.9.A Evaluate logarithmic expressions.

ESSENTIAL KNOWLEDGE

- **2.9.A.1** The logarithmic expression $\log_b c$ is equal to, or represents, the value that the base b must be exponentially raised to in order to obtain the value c. That is, $\log_b c = a$ if and only if $b^a = c$, where a and c are constants, $b > 0$, and $b \neq 1$. (when the base of a logarithmic expression is not specified, it is understood as the common logarithm with a base of 10)
- **2.9.A.2** The values of some logarithmic expressions are readily accessible through basic arithmetic while other values can be estimated through the use of technology.
- **2.9.A.3** On a logarithmic scale, each unit represents a multiplicative change of the base of the logarithm. For example, on a standard scale, the units might be $0, 1, 2, \ldots$, while on a logarithmic scale, using log base 10, the units might be $10^0, 10^1, 10^2, \ldots$.

TOPIC 2.10

Inverses of Exponential Functions

Instructional Periods: 2
Skills Focus: 1.C, 2.B

LEARNING OBJECTIVES

2.10.A Construct representations of the inverse of an exponential function with an initial value of 1.

ESSENTIAL KNOWLEDGE

- **2.10.A.1** The general form of a logarithmic function is $f(x)=a\log_b x$, with *base* b, where $b>0$, $b\neq 1$, and $a\neq 0$.
- **2.10.A.2** The way in which input and output values vary together have an inverse relationship in exponential and logarithmic functions. Output values of general-form exponential functions change proportionately as input values increase in equal-length intervals. However, input values of general-form logarithmic functions change proportionately as output values increase in equal-length intervals. Alternately, exponential growth is characterized by output values changing multiplicatively as input values change additively, whereas logarithmic growth is characterized by output values changing additively as input values change multiplicatively.
- **2.10.A.3** $f(x)=\log_b x$ and $g(x)=b^x$, where $b>0$ and $b\neq 1$, are inverse functions. That is, $g(f(x))=f(g(x))=x$.
- **2.10.A.4** The graph of the logarithmic function $f(x)=\log_b x$, where $b>0$ and $b\neq 1$, is a reflection of the graph of the exponential function $g(x)=b^x$, where $b>0$ and $b\neq 1$, over the graph of the identity function $h(x)=x$.
- **2.10.A.5** If (s,t) is an ordered pair of the exponential function $g(x)=b^x$, where $b>0$ and $b\neq 1$, then (t,s) is an ordered pair of the logarithmic function $f(x)=\log_b x$, where $b>0$ and $b\neq 1$.

TOPIC 2.11

Logarithmic Functions

Instructional Periods: 1
Skills Focus: 3.A

LEARNING OBJECTIVES

2.11.A Identify key characteristics of logarithmic functions.

ESSENTIAL KNOWLEDGE

- **2.11.A.1** The domain of a logarithmic function in general form is any real number greater than zero, and its range is all real numbers.
- **2.11.A.2** Because logarithmic functions are inverses of exponential functions, logarithmic functions are also always increasing or always decreasing, and their graphs are either always concave up or always concave down. Consequently, logarithmic functions do not have extrema except on a closed interval, and their graphs do not have inflection points.
- **2.11.A.3** The additive transformation function $g(x)=f(x+k)$, where $k\neq 0$, of a logarithmic function f in general form does not have input values that are proportional over equal-length output-value intervals. However, if the output values of the additive transformation function, $g(x)=f(x+k)$, of any function f are proportional over equal-length input-value intervals, then f is logarithmic.
- **2.11.A.4** With their limited domain, logarithmic functions in general form are vertically asymptotic to $x=0$, with an end behavior that is unbounded. That is, for a logarithmic function in general form, $\lim_{x\to 0^+} a\log_b x=\pm\infty$ and $\lim_{x\to\infty} a\log_b x=\pm\infty$.

TOPIC 2.12

Logarithmic Function Manipulation

Instructional Periods: 2
Skills Focus: 1.B, 3.A

LEARNING OBJECTIVES

2.12.A Rewrite logarithmic expressions in equivalent forms.

ESSENTIAL KNOWLEDGE

- **2.12.A.1** The product property for logarithms states that $\log_b(xy) = \log_b x + \log_b y$. Graphically, this property implies that every horizontal dilation of a logarithmic function, $f(x) = \log_b(kx)$, is equivalent to a vertical translation, $f(x) = \log_b(kx) = \log_b k + \log_b x = a + \log_b x$, where $a = \log_b k$.
- **2.12.A.2** The power property for logarithms states that $\log_b x^n = n\log_b x$. Graphically, this property implies that raising the input of a logarithmic function to a power, $f(x) = \log_b x^k$, results in a vertical dilation, $f(x) = \log_b x^k = k\log_b x$.
- **2.12.A.3** The change of base property for logarithms states that $\log_b x = \frac{\log_a x}{\log_a b}$, where $a > 0$ and $a \neq 1$. This implies that all logarithmic functions are vertical dilations of each other.
- **2.12.A.4** The function $f(x) = \ln x$ is a logarithmic function with the natural base e; that is, $\ln x = \log_e x$.

TOPIC 2.13

Exponential and Logarithmic Equations and Inequalities

Instructional Periods: 3
Skills Focus: 1.A, 1.B, 1.C

LEARNING OBJECTIVES

2.13.A Solve exponential and logarithmic equations and inequalities.

ESSENTIAL KNOWLEDGE

- **2.13.A.1** Properties of exponents, properties of logarithms, and the inverse relationship between exponential and logarithmic functions can be used to solve equations and inequalities involving exponents and logarithms.
- **2.13.A.2** When solving exponential and logarithmic equations found through analytical or graphical methods, the results should be examined for extraneous solutions precluded by the mathematical or contextual limitations.
- **2.13.A.3** Logarithms can be used to rewrite expressions involving exponential functions in different ways that may reveal helpful information. Specifically, $b^x = c^{(\log_c b)(x)}$.

LEARNING OBJECTIVES

2.13.B Construct the inverse function for exponential and logarithmic functions.

ESSENTIAL KNOWLEDGE

- **2.13.B.1** The function $f(x) = ab^{(x-h)} + k$ is a combination of additive transformations of an exponential function in general form. The inverse of $y = f(x)$ can be found by determining the inverse operations to reverse the mapping.
- **2.13.B.2** The function $f(x) = a\log_b(x-h)+k$ is a combination of additive transformations of a logarithmic function in general form. The inverse of $y = f(x)$ can be found by determining the inverse operations to reverse the mapping.

TOPIC 2.14

Logarithmic Function Context and Data Modeling

Instructional Periods: 2
Skills Focus: 1.C, 3.B

LEARNING OBJECTIVES

2.14.A Construct a logarithmic function model.

ESSENTIAL KNOWLEDGE

- **2.14.A.1** Logarithmic functions are inverses of exponential functions and can be used to model situations involving proportional growth, or repeated multiplication, where the input values change proportionally over equal-length output-value intervals. Alternately, if the output value is a whole number, it indicates how many times the initial value has been multiplied by the proportion.
- **2.14.A.2** A logarithmic function model can be constructed from an appropriate proportion and a real zero or from two input-output pairs.
- **2.14.A.3** Logarithmic function models can be constructed by applying transformations to $f(x)=a\log_b x$ based on characteristics of a context or data set.
- **2.14.A.4** Logarithmic function models can be constructed for a data set with technology using logarithmic regressions.
- **2.14.A.5** The natural log function is often useful in modeling natural phenomena.
- **2.14.A.6** Logarithmic function models can be used to predict values for the dependent variable.

TOPIC 2.15

Semi-log Plots

Instructional Periods: 2
Skills Focus: 2.B, 3.C

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
2.15.A Determine if an exponential model is appropriate by examining a semi-log plot of a data set.	▪ **2.15.A.1** In a semi-log plot, one of the axes is logarithmically scaled. When the *y*-axis of a semi-log plot is logarithmically scaled, data or functions that demonstrate exponential characteristics will appear linear. ▪ **2.15.A.2** An advantage of semi-log plots is that a constant never needs to be added to the dependent variable values to reveal that an exponential model is appropriate.
2.15.B Construct the linearization of exponential data.	▪ **2.15.B.1** Techniques used to model linear functions can be applied to a semi-log graph. ▪ **2.15.B.2** For an exponential model of the form $y = ab^x$, the corresponding linear model for the semi-log plot is $y = (\log_n b)x + \log_n a$, where $n > 0$ and $n \neq 1$. Specifically, the linear rate of change is $\log_n b$, and the initial linear value is $\log_n a$.

UNIT 3:

Trigonometric and Polar Functions

7–7.5 WEEKS

Unit at a Glance

Topic #	Topic Title	Instructional Periods	Suggested Skill Focus
3.1	Periodic Phenomena	2	2.B, 3.A
3.2	Sine, Cosine, and Tangent	2	2.A, 3.A
3.3	Sine and Cosine Function Values	2	2.A, 3.B
3.4	Sine and Cosine Function Graphs	2	2.A, 3.A
3.5	Sinusoidal Functions	2	2.A, 3.A
3.6	Sinusoidal Function Transformations	2	1.C, 2.B
3.7	Sinusoidal Function Context and Data Modeling	2	1.C, 3.C
3.8	The Tangent Function	2	2.A, 3.A
3.9	Inverse Trigonometric Functions	2	1.C, 2.B
3.10	Trigonometric Equations and Inequalities	3	1.A, 2.A, 3.B
3.11	The Secant, Cosecant, and Cotangent Functions	2	2.B, 3.A
3.12	Equivalent Representations of Trigonometric Functions	3	1.A, 1.B, 3.B
3.13	Trigonometry and Polar Coordinates	2	1.B, 2.A
3.14	Polar Function Graphs	2	2.B, 3.A
3.15	Rates of Change in Polar Functions	2	3.A, 3.C

TOPIC 3.1

Periodic Phenomena

Instructional Periods: 2
Skills Focus: 2.B, 3.A

LEARNING OBJECTIVES

3.1.A Construct graphs of periodic relationships based on verbal representations.

3.1.B Describe key characteristics of a periodic function based on a verbal representation.

ESSENTIAL KNOWLEDGE

- **3.1.A.1** A periodic relationship can be identified between two aspects of a context if, as the input values increase, the output values demonstrate a repeating pattern over successive equal-length intervals.
- **3.1.A.2** The graph of a periodic relationship can be constructed from the graph of a single cycle of the relationship.
- **3.1.B.1** The *period* of the function is the smallest positive value k such that $f(x+k)=f(x)$ for all x in the domain. Consequently, the behavior of a periodic function is completely determined by any interval of width k.
- **3.1.B.2** The period can be estimated by investigating successive equal-length output values and finding where the pattern begins to repeat.
- **3.1.B.3** Periodic functions take on characteristics of other functions, such as intervals of increase and decrease, different concavities, and various rates of change. However, with periodic functions, all characteristics found in one period of the function will be in every period of the function.

TOPIC 3.2

Sine, Cosine, and Tangent

Instructional Periods: 2
Skills Focus: 2.A, 3.A

LEARNING OBJECTIVES

3.2.A Determine the sine, cosine, and tangent of an angle using the unit circle.

ESSENTIAL KNOWLEDGE

- **3.2.A.1** In the coordinate plane, an angle is in *standard position* when the vertex coincides with the origin and one ray coincides with the positive *x*-axis. The other ray is called the *terminal ray*. Positive and negative angle measures indicate rotations from the positive *x*-axis in the counterclockwise and clockwise direction, respectively. Angles in standard position that share a terminal ray differ by an integer number of revolutions.
- **3.2.A.2** The radian measure of an angle in standard position is the ratio of the length of the arc of a circle centered at the origin subtended by the angle to the radius of that same circle. For a unit circle, which has radius 1, the radian measure is the same as the length of the subtended arc.
- **3.2.A.3** Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The *sine* of the angle is the ratio of the vertical displacement of P from the *x*-axis to the distance between the origin and point P. Therefore, for a unit circle, the sine of the angle is the *y*-coordinate of point P.
- **3.2.A.4** Given an angle in standard position and a circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The *cosine* of the angle is the ratio of the horizontal displacement of P from the *y*-axis to the distance between the origin and point P. Therefore, for a unit circle, the cosine of the angle is the *x*-coordinate of point P.
- **3.2.A.5** Given an angle in standard position, the *tangent* of the angle is the slope, if it exists, of the terminal ray. Because the slope of the terminal ray is the ratio of the vertical displacement to the horizontal displacement over any interval, the tangent of the angle is the ratio of the *y*-coordinate to the *x*-coordinate of the point at which the terminal ray intersects the unit circle; alternately, it is the ratio of the angle's sine to its cosine.

TOPIC 3.3

Sine and Cosine Function Values

Instructional Periods: 2
Skills Focus: 2.A, 3.B

LEARNING OBJECTIVES

3.3.A Determine coordinates of points on a circle centered at the origin.

ESSENTIAL KNOWLEDGE

- **3.3.A.1** Given an angle of measure θ in standard position and a circle with radius r centered at the origin, there is a point, P, where the terminal ray intersects the circle. The coordinates of point P are $\left(r\cos\theta, r\sin\theta\right)$.
- **3.3.A.2** The geometry of isosceles right and equilateral triangles, while attending to the signs of the values based on the quadrant of the angle, can be used to find exact values for the cosine and sine of angles that are multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$ radians and whose terminal rays do not lie on an axis.

TOPIC 3.4

Sine and Cosine Function Graphs

Instructional Periods: 2
Skills Focus: 2.A, 3.A

LEARNING OBJECTIVES

3.4.A Construct representations of the sine and cosine functions using the unit circle.

ESSENTIAL KNOWLEDGE

- **3.4.A.1** Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The sine function, $f(\theta) = \sin\theta$, gives the *y*-coordinate, or vertical displacement from the *x*-axis, of point P. The domain of the sine function is all real numbers.
- **3.4.A.2** As the input values, or angles, of the sine function increase, the output values oscillate between negative one and one, taking every value in between and tracking the vertical distance of points on the unit circle from the *x*-axis.
- **3.4.A.3** Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The cosine function, $f(\theta) = \cos\theta$, gives the *x*-coordinate, or horizontal displacement from the *y*-axis, of point P. The domain of the cosine function is all real numbers.
- **3.4.A.4** As the input values, or angles, of the cosine function increase, the output values oscillate between negative one and one, taking every value in between and tracking the horizontal distance of points on the unit circle from the *y*-axis.

TOPIC 3.5

Sinusoidal Functions

Instructional Periods: 2
Skills Focus: 2.A, 3.A

LEARNING OBJECTIVES

3.5.A Identify key characteristics of the sine and cosine functions.

ESSENTIAL KNOWLEDGE

- **3.5.A.1** A *sinusoidal function* is any function that involves additive and multiplicative transformations of $f(\theta)=\sin\theta$. The sine and cosine functions are both sinusoidal functions, with $\cos\theta=\sin\left(\theta+\frac{\pi}{2}\right)$.
- **3.5.A.2** The period and frequency of a sinusoidal function are reciprocals. The period of $f(\theta)=\sin\theta$ and $g(\theta)=\cos\theta$ is 2π, and the frequency is $\frac{1}{2\pi}$.
- **3.5.A.3** The amplitude of a sinusoidal function is half the difference between its maximum and minimum values. The amplitude of $f(\theta)=\sin\theta$ and $g(\theta)=\cos\theta$ is 1.
- **3.5.A.4** The midline of the graph of a sinusoidal function is determined by the average, or arithmetic mean, of the maximum and minimum values of the function. The midline of the graphs of $y=\sin\theta$ and $y=\cos\theta$ is $y=0$.
- **3.5.A.5** As input values increase, the graphs of sinusoidal functions oscillate between concave down and concave up.
- **3.5.A.6** The graph of $y=\sin\theta$ has rotational symmetry about the origin and is therefore an odd function. The graph of $y=\cos\theta$ has reflective symmetry over the *y*-axis and is therefore an even function.

TOPIC 3.6

Sinusoidal Function Transformations

Instructional Periods: 2
Skills Focus: 1.C, 2.B

LEARNING OBJECTIVES

3.6.A Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.

ESSENTIAL KNOWLEDGE

- **3.6.A.1** Functions that can be written in the form $f(\theta)=a\sin(b(\theta+c))+d$ or $g(\theta)=a\cos(b(\theta+c))+d$, where a, b, c, and d are real numbers and $a\neq 0$, are sinusoidal functions and are transformations of the sine and cosine functions. Additive and multiplicative transformations are the same for both sine and cosine, because the cosine function is a phase shift of the sine function by $-\frac{\pi}{2}$ units.
- **3.6.A.2** The graph of the additive transformation $g(\theta)=\sin\theta+d$ of the sine function $f(\theta)=\sin\theta$ is a vertical translation of the graph of f, including its midline, by d units. The same transformation of the cosine function yields the same result.
- **3.6.A.3** The graph of the additive transformation $g(\theta)=\sin(\theta+c)$ of the sine function $f(\theta)=\sin\theta$ is a horizontal translation, or phase shift, of the graph of f by $-c$ units. The same transformation of the cosine function yields the same result.
- **3.6.A.4** The graph of the multiplicative transformation $g(\theta)=a\sin\theta$ of the sine function $f(\theta)=\sin\theta$ is a vertical dilation of the graph of f and differs in amplitude by a factor of $|a|$. The same transformation of the cosine function yields the same result.
- **3.6.A.5** The graph of the multiplicative transformation $g(\theta)=\sin(b\theta)$ of the sine function $f(\theta)=\sin\theta$ is a horizontal dilation of the graph of f and differs in period by a factor of $\left|\frac{1}{b}\right|$. The same transformation of the cosine function yields the same result.

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
3.6.A Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	▪ **3.6.A.6** The graph of $y = f(\theta) = a\sin(b(\theta + c)) + d$ has an amplitude of $\lvert a \rvert$ units, a period of $\left\lvert \frac{1}{b} \right\rvert 2\pi$ units, a midline vertical shift of d units from $y = 0$, and a phase shift of $-c$ units. The same transformations of the cosine function yield the same results.

TOPIC 3.7

Sinusoidal Function Context and Data Modeling

Instructional Periods: 2
Skills Focus: 1.C, 3.C

LEARNING OBJECTIVES

3.7.A Construct sinusoidal function models of periodic phenomena by estimating key values.

ESSENTIAL KNOWLEDGE

- **3.7.A.1** The smallest interval of input values over which the maximum or minimum output values start to repeat can be used to determine or estimate the period and frequency for a sinusoidal function model.
- **3.7.A.2** The maximum and minimum output values can be used to determine or estimate the amplitude and vertical shift for a sinusoidal function model.
- **3.7.A.3** An actual pair of input-output values can be compared to pairs of input-output values produced by a sinusoidal function model to determine or estimate a phase shift for the model.
- **3.7.A.4** Technology can be used to find an appropriate sinusoidal function model for a data set.
- **3.7.A.5** Sinusoidal functions that model a data set are frequently only useful over their contextual domain and can be used to predict values of the dependent variable from a value of the independent variable.

TOPIC 3.8

The Tangent Function

Instructional Periods: 2
Skills Focus: 2.A, 3.A

LEARNING OBJECTIVES

3.8.A Construct representations of the tangent function using the unit circle.

ESSENTIAL KNOWLEDGE

- **3.8.A.1** Given an angle of measure θ in standard position and a unit circle centered at the origin, there is a point, P, where the terminal ray intersects the circle. The tangent function, $f(\theta) = \tan\theta$, gives the slope of the terminal ray.
- **3.8.A.2** Because the slope of the terminal ray is the ratio of the change in the *y*-values to the change in the *x*-values between any two points on the ray, the tangent function is also the ratio of the sine function to the cosine function. Therefore, $\tan\theta = \frac{\sin\theta}{\cos\theta}$, where $\cos\theta \neq 0$.

LEARNING OBJECTIVES

3.8.B Describe key characteristics of the tangent function.

ESSENTIAL KNOWLEDGE

- **3.8.B.1** Because the slope values of the terminal ray repeat every one-half revolution of the circle, the tangent function has a period of π.
- **3.8.B.2** The tangent function demonstrates periodic asymptotic behavior at input values $\theta = \frac{\pi}{2} + k\pi$, for integer values of k, because $\cos\theta = 0$ at those values.
- **3.8.B.3** The tangent function increases and its graph changes from concave down to concave up between consecutive asymptotes.

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
3.8.C Describe additive and multiplicative transformations involving the tangent function.	▪ **3.8.C.1** The graph of the additive transformation $g(\theta)=\tan\theta+d$ of the tangent function $f(\theta)=\tan\theta$ is a vertical translation of the graph of f and the line containing its inflection points by d units. ▪ **3.8.C.2** The graph of the additive transformation $g(\theta)=\tan(\theta+c)$ of the tangent function $f(\theta)=\tan\theta$ is a horizontal translation, or phase shift, of the graph of f by $-c$ units. ▪ **3.8.C.3** The graph of the multiplicative transformation $g(\theta)=a\tan\theta$ of the tangent function $f(\theta)=\tan\theta$ is a vertical dilation of the graph of f by a factor of $\lvert a\rvert$. If $a<0$, the transformation involves a reflection over the *x*-axis. ▪ **3.8.C.4** The graph of the multiplicative transformation $g(\theta)=\tan(b\theta)$ of the tangent function $f(\theta)=\tan\theta$ is a horizontal dilation of the graph of f and differs in period by a factor of $\left\lvert\frac{1}{b}\right\rvert$. If $b<0$, the transformation involves a reflection over the *y*-axis. ▪ **3.8.C.5** The graph of $y=f(\theta)=a\tan(b(\theta+c))+d$ is a vertical dilation of the graph of $y=\tan\theta$ by a factor of $\lvert a\rvert$, has a period of $\left\lvert\frac{1}{b}\right\rvert\pi$ units, is a vertical shift of the line containing the inflection points of the graph of $y=\tan\theta$ by d units, and is a phase shift of $-c$ units.

TOPIC 3.9

Inverse Trigonometric Functions

Instructional Periods: 2
Skills Focus: 1.C, 2.B

LEARNING OBJECTIVES

3.9.A Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain.

ESSENTIAL KNOWLEDGE

- **3.9.A.1** For inverse trigonometric functions, the input and output values are switched from their corresponding trigonometric functions, so the output value of an inverse trigonometric function is often interpreted as an angle measure and the input is a value in the range of the corresponding trigonometric function.
- **3.9.A.2** The inverse trigonometric functions are called *arcsine*, *arccosine*, and *arctangent* (also represented as $\sin^{-1} x$, $\cos^{-1} x$, and $\tan^{-1} x$). Because the corresponding trigonometric functions are periodic, they are only invertible if they have restricted domains.
- **3.9.A.3** In order to define their respective inverse functions, the domain of the sine function is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the cosine function to $[0, \pi]$, and the tangent function to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

TOPIC 3.10

Trigonometric Equations and Inequalities

Instructional Periods: 3
Skills Focus: 1.A, 2.A, 3.B

LEARNING OBJECTIVES

3.10.A Solve equations and inequalities involving trigonometric functions.

ESSENTIAL KNOWLEDGE

- **3.10.A.1** Inverse trigonometric functions are useful in solving equations and inequalities involving trigonometric functions, but solutions may need to be modified due to domain restrictions.
- **3.10.A.2** Because trigonometric functions are periodic, there are often infinite solutions to trigonometric equations.
- **3.10.A.3** In trigonometric equations and inequalities arising from a contextual scenario, there is often a domain restriction that can be implied from the context, which limits the number of solutions.

TOPIC 3.11

The Secant, Cosecant, and Cotangent Functions

Instructional Periods: 2
Skills Focus: 2.B, 3.A

LEARNING OBJECTIVES

3.11.A Identify key characteristics of functions that involve quotients of the sine and cosine functions.

ESSENTIAL KNOWLEDGE

- **3.11.A.1** The secant function, $f(\theta) = \sec\theta$, is the reciprocal of the cosine function, where $\cos\theta \neq 0$.
- **3.11.A.2** The cosecant function, $f(\theta) = \csc\theta$, is the reciprocal of the sine function, where $\sin\theta \neq 0$.
- **3.11.A.3** The graphs of the secant and cosecant functions have vertical asymptotes where cosine and sine are zero, respectively, and have a range of $(-\infty, -1] \cup [1, \infty)$.
- **3.11.A.4** The cotangent function, $f(\theta) = \cot\theta$, is the reciprocal of the tangent function, where $\tan\theta \neq 0$. Equivalently, $\cot\theta = \frac{\cos\theta}{\sin\theta}$, where $\sin\theta \neq 0$.
- **3.11.A.5** The graph of the cotangent function has vertical asymptotes for domain values where $\tan\theta = 0$ and is decreasing between consecutive asymptotes.

TOPIC 3.12

Equivalent Representations of Trigonometric Functions

Instructional Periods: 3
Skills Focus: 1.A, 1.B, 3.B

LEARNING OBJECTIVES

3.12.A Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.

ESSENTIAL KNOWLEDGE

- **3.12.A.1** The Pythagorean theorem can be applied to right triangles with points on the unit circle at coordinates $(\cos\theta, \sin\theta)$, resulting in the Pythagorean identity: $\sin^2\theta + \cos^2\theta = 1$.
- **3.12.A.2** The Pythagorean identity can be algebraically manipulated into other forms involving trigonometric functions, such as $\tan^2\theta = \sec^2\theta - 1$, and can be used to establish other trigonometric relationships, such as $\arcsin x = \arccos\sqrt{1-x^2}$, with appropriate domain restrictions.

LEARNING OBJECTIVES

3.12.B Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities.

ESSENTIAL KNOWLEDGE

- **3.12.B.1** The sum identity for sine is $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.
- **3.12.B.2** The sum identity for cosine is $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.
- **3.12.B.3** The sum identities for sine and cosine can also be used as difference and double-angle identities.
- **3.12.B.4** Properties of trigonometric functions, known trigonometric identities, and other algebraic properties can be used to verify additional trigonometric identities.

LEARNING OBJECTIVES

3.12.C Solve equations using equivalent analytic representations of trigonometric functions.

ESSENTIAL KNOWLEDGE

- **3.12.C.1** A specific equivalent form involving trigonometric expressions can make information more accessible.
- **3.12.C.2** Equivalent trigonometric forms may be useful in solving trigonometric equations and inequalities.

TOPIC 3.13

Trigonometry and Polar Coordinates

Instructional Periods: 2
Skills Focus: 1.B, 2.A

LEARNING OBJECTIVES

3.13.A Determine the location of a point in the plane using both rectangular and polar coordinates.

ESSENTIAL KNOWLEDGE

- **3.13.A.1** The polar coordinate system is based on a grid of circles centered at the origin and on lines through the origin. Polar coordinates are defined as an ordered pair, (r, θ), such that $|r|$ represents the radius of the circle on which the point lies, and θ represents the measure of an angle in standard position whose terminal ray includes the point. In the polar coordinate system, the same point can be represented many ways.
- **3.13.A.2** The coordinates of a point in the polar coordinate system, (r, θ), can be converted to coordinates in the rectangular coordinate system, (x, y), using $x = r\cos\theta$ and $y = r\sin\theta$.
- **3.13.A.3** The coordinates of a point in the rectangular coordinate system, (x, y), can be converted to coordinates in the polar coordinate system, (r, θ), using $r = \sqrt{x^2 + y^2}$ and $\theta = \arctan\frac{y}{x}$ for $x > 0$ or $\theta = \arctan\frac{y}{x} + \pi$ for $x < 0$.
- **3.13.A.4** A complex number can be understood as a point in the complex plane and can be determined by its corresponding rectangular or polar coordinates. When the complex number has the rectangular coordinates (a,b), it can be expressed as $a + bi$. When the complex number has polar coordinates (r,θ), it can be expressed as $(r\cos\theta) + i(r\sin\theta)$.

TOPIC 3.14

Polar Function Graphs

Instructional Periods: 2
Skills Focus: 2.B, 3.A

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
3.14.A Construct graphs of polar functions.	▪ **3.14.A.1** The graph of the function $r = f(\theta)$ in polar coordinates consists of input-output pairs of values where the input values are angle measures and the output values are radii. ▪ **3.14.A.2** The domain of the polar function $r = f(\theta)$, given graphically, can be restricted to a desired portion of the function by selecting endpoints corresponding to the desired angle and radius. ▪ **3.14.A.3** When graphing polar functions in the form of $r = f(\theta)$, changes in input values correspond to changes in angle measure from the positive *x*-axis, and changes in output values correspond to changes in distance from the origin.

TOPIC 3.15

Rates of Change in Polar Functions

Instructional Periods: 2
Skills Focus: 3.A, 3.C

LEARNING OBJECTIVES

3.15.A Describe characteristics of the graph of a polar function.

ESSENTIAL KNOWLEDGE

- **3.15.A.1** If a polar function, $r = f(\theta)$, is positive and increasing or negative and decreasing, then the distance between $f(\theta)$ and the origin is increasing.
- **3.15.A.2** If a polar function, $r = f(\theta)$, is positive and decreasing or negative and increasing, then the distance between $f(\theta)$ and the origin is decreasing.
- **3.15.A.3** For a polar function, $r = f(\theta)$, if the function changes from increasing to decreasing or decreasing to increasing on an interval, then the function has a relative extremum on the interval corresponding to a point relatively closest to or farthest from the origin.
- **3.15.A.4** The average rate of change of r with respect to θ over an interval of θ is the ratio of the change in the radius values to the change in θ over an interval of θ. Graphically, the average rate of change indicates the rate at which the radius is changing per radian.
- **3.15.A.5** The average rate of change of r with respect to θ over an interval of θ can be used to estimate values of the function within the interval.

UNIT 4:

Functions Involving Parameters, Vectors, and Matrices

7–7.5 WEEKS

Unit at a Glance

Topic #	Topic Title	Instructional Periods	Suggested Skill Focus
4.1	Parametric Functions	2	1.A, 2.B
4.2	Parametric Functions Modeling Planar Motion	2	3.A, 3.B
4.3	Parametric Functions and Rates of Change	2	3.B, 3.C
4.4	Parametrically Defined Circles and Lines	2	1.B, 1.C
4.5	Implicitly Defined Functions	2	2.B, 3.A
4.6	Conic Sections	3	1.B, 2.A, 2.B
4.7	Parametrization of Implicitly Defined Functions	2	1.B, 2.A
4.8	Vectors	3	2.A, 3.A, 3.B
4.9	Vector-Valued Functions	1	3.C
4.10	Matrices	2	1.B, 3.B
4.11	The Inverse and Determinant of a Matrix	2	1.B, 3.B
4.12	Linear Transformations and Matrices	1	1.B
4.13	Matrices as Functions	3	1.B, 2.A, 3.A
4.14	Matrices Modeling Contexts	3	1.C, 3.B, 3.C

TOPIC 4.1

Parametric Functions

Instructional Periods: 2
Skills Focus: 1.A, 2.B

LEARNING OBJECTIVES

4.1.A Construct a graph or table of values for a parametric function represented analytically.

ESSENTIAL KNOWLEDGE

- **4.1.A.1** A parametric function in R^2, the set of all ordered pairs of two real numbers, consists of a set of two parametric equations in which two dependent variables, x and y, are dependent on a single independent variable, t, called the *parameter*.
- **4.1.A.2** Because variables x and y are dependent on the independent variable, t, the coordinates (x_i, y_i) at time t_i can be written as functions of t and can be expressed as the single parametric function $f(t)=(x(t), y(t))$, where in this case x and y are names of two functions.
- **4.1.A.3** A numerical table of values can be generated for the parametric function $f(t)=(x(t), y(t))$ by evaluating x_i and y_i at several values of t_i within the domain.
- **4.1.A.4** A graph of a parametric function can be sketched by connecting several points from the numerical table of values in order of increasing value of t.
- **4.1.A.5** The domain of the parametric function f is often restricted, which results in start and end points on the graph of f.

TOPIC 4.2

Parametric Functions Modeling Planar Motion

Instructional Periods: 2
Skills Focus: 3.A, 3.B

LEARNING OBJECTIVES

4.2.A Identify key characteristics of a parametric planar motion function that are related to position.

ESSENTIAL KNOWLEDGE

- **4.2.A.1** A parametric function given by $f(t) = (x(t), y(t))$ can be used to model particle motion in the plane. The graph of this function indicates the position of a particle at time t.
- **4.2.A.2** The horizontal and vertical extrema of a particle's motion can be determined by identifying the maximum and minimum values of the functions $x(t)$ and $y(t)$, respectively.
- **4.2.A.3** The real zeros of the function $x(t)$ correspond to *y*-intercepts, and the real zeros of $y(t)$ correspond to *x*-intercepts.

TOPIC 4.3

Parametric Functions and Rates of Change

Instructional Periods: 2
Skills Focus: 3.B, 3.C

LEARNING OBJECTIVES

4.3.A Identify key characteristics of a parametric planar motion function that are related to direction and rate of change.

ESSENTIAL KNOWLEDGE

- **4.3.A.1** As the parameter increases, the direction of planar motion of a particle can be analyzed in terms of x and y independently. If $x(t)$ is increasing or decreasing, the direction of motion is to the right or left, respectively. If $y(t)$ is increasing or decreasing, the direction of motion is up or down, respectively.
- **4.3.A.2** At any given point in the plane, the direction of planar motion may be different for different values of t.
- **4.3.A.3** The same curve in the plane can be parametrized in different ways and can be traversed in different directions with different parametric functions.
- **4.3.A.4** Over a given interval $[t_1, t_2]$ within the domain, the average rate of change can be computed for $x(t)$ and $y(t)$ independently. The ratio of the average rate of change of y to the average rate of change of x gives the slope of the graph between the points on the curve corresponding to t_1 and t_2, so long as the average rate of change of $x(t) \neq 0$.

TOPIC 4.4

Parametrically Defined Circles and Lines

Instructional Periods: 2
Skills Focus: 1.B, 1.C

LEARNING OBJECTIVES

4.4.A Express motion around a circle or along a line segment parametrically.

ESSENTIAL KNOWLEDGE

- **4.4.A.1** A complete counterclockwise revolution around the unit circle that starts and ends at $(1,0)$ and is centered at the origin can be modeled by $(x(t), y(t)) = (\cos t, \sin t)$ with domain $0 \le t \le 2\pi$.
- **4.4.A.2** Transformations of the parametric function $(x(t), y(t)) = (\cos t, \sin t)$ can model any circular path traversed in the plane.
- **4.4.A.3** A linear path along the line segment from the point (x_1, y_1) to the point (x_2, y_2) can be parametrized many ways, including using an initial position (x_1, y_1) and rates of change for x with respect to t and y with respect to t.

TOPIC 4.5

Implicitly Defined Functions

Instructional Periods: 2
Skills Focus: 2.B, 3.A

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
4.5.A Construct a graph of an equation involving two variables.	▪ **4.5.A.1** An equation involving two variables can implicitly describe one or more functions. ▪ **4.5.A.2** An equation involving two variables can be graphed by finding solutions to the equation. ▪ **4.5.A.3** Solving for one of the variables in an equation involving two variables can define a function whose graph is part or all of the graph of the equation.
4.5.B Determine how the two quantities related in an implicitly defined function vary together.	▪ **4.5.B.1** For ordered pairs on the graph of an implicitly defined function that are close together, if the ratio of the change in the two variables is positive, then the two variables simultaneously increase or both decrease; conversely, if the ratio is negative, then as one variable increases, the other decreases. ▪ **4.5.B.2** The rate of change of x with respect to y or of y with respect to x can be zero, indicating vertical or horizontal intervals, respectively.

TOPIC 4.6

Conic Sections

Instructional Periods: 3
Skills Focus: 1.B, 2.A, 2.B

LEARNING OBJECTIVES

4.6.A Represent conic sections with horizontal or vertical symmetry analytically.

ESSENTIAL KNOWLEDGE

- **4.6.A.1** A parabola with vertex (h, k) can, if $a \neq 0$, be represented analytically as $(y-k)^2 = a(x-h)$ if it opens left or right, or as $a(y-k) = (x-h)^2$ if it opens up or down.
- **4.6.A.2** An ellipse centered at (h, k) with horizontal radius a and vertical radius b can be represented analytically as $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$. A circle is a special case of an ellipse where $a = b$.
- **4.6.A.3** A hyperbola centered at (h, k) with vertical and horizonal lines of symmetry can be represented algebraically as $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ for a hyperbola opening left and right, or as $-\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ for a hyperbola opening up and down.

TOPIC 4.7

Parametrization of Implicitly Defined Functions

Instructional Periods: 2
Skills Focus: 1.B, 2.A

LEARNING OBJECTIVES

4.7.A Represent a curve in the plane parametrically.

4.7.B Represent conic sections parametrically.

ESSENTIAL KNOWLEDGE

- **4.7.A.1** A parametrization $(x(t), y(t))$ for an implicitly defined function will, when $x(t)$ and $y(t)$ are substituted for x and y, respectively, satisfy the corresponding equation for every value of t in the domain.
- **4.7.A.2** If f is a function of x, then $y=f(x)$ can be parametrized as $(x(t), y(t))=(t, f(t))$. If f is invertible, its inverse can be parametrized as $(x(t), y(t))=(f(t), t)$ for an appropriate interval of t.

- **4.7.B.1** A parabola can be parametrized in the same way that any equation that can be solved for x or y can be parametrized. Equations that can be solved for x can be parametrized as $(x(t), y(t))=(f(t), t)$ by solving for x and replacing y with t. Equations that can be solved for y can be parametrized as $(x(t), y(t))=(t, f(t))$ by solving for y and replacing x with t.
- **4.7.B.2** An ellipse can be parametrized using the trigonometric functions $x(t)=h+a\cos t$ and $y(t)=k+b\sin t$ for $0 \le t \le 2\pi$.
- **4.7.B.3** A hyperbola can be parametrized using trigonometric functions. For a hyperbola that opens left and right, the functions are $x(t)=h+a\sec t$ and $y(t)=k+b\tan t$ for $0 \le t \le 2\pi$. For a hyperbola that opens up and down, the functions are $x(t)=h+a\tan t$ and $y(t)=k+b\sec t$ for $0 \le t \le 2\pi$.

TOPIC 4.8

Vectors

Instructional Periods: 3
Skills Focus: 2.A, 3.A, 3.B

LEARNING OBJECTIVES

4.8.A Identify characteristics of a vector.

ESSENTIAL KNOWLEDGE

- **4.8.A.1** A vector is a directed line segment. When a vector is placed in the plane, the point at the beginning of the line segment is called the *tail*, and the point at the end of the line segment is called the *head*. The length of the line segment is the *magnitude* of the vector.
- **4.8.A.2** A vector $\overrightarrow{P_1P_2}$ with two components can be plotted in the *xy*-plane from $P_1=(x_1,y_1)$ to $P_2=(x_2,y_2)$. The vector is identified by a and b, where $a=x_2-x_1$ and $b=y_2-y_1$. The vector can be expressed as $\langle a,b\rangle$. A zero vector $\langle 0,0\rangle$ is the trivial case when $P_1=P_2$.
- **4.8.A.3** The *direction* of the vector is parallel to the line segment from the origin to the point with coordinates (a,b). The magnitude of the vector is the square root of the sum of the squares of the components.
- **4.8.A.4** For a vector represented geometrically in the plane, the components of the vector can be found using trigonometry.

LEARNING OBJECTIVES

4.8.B Determine sums and products involving vectors.

ESSENTIAL KNOWLEDGE

- **4.8.B.1** The multiplication of a constant and a vector results in a new vector whose components are found by multiplying the constant by each of the components of the original vector. The new vector is parallel to the original vector.
- **4.8.B.2** The sum of two vectors in R^2 is a new vector whose components are found by adding the corresponding components of the original vectors. The new vector can be represented graphically as a vector whose tail corresponds to the tail of the first vector and whose head corresponds to the head of the second vector when the second vector's tail is located at the first vector's head.
- **4.8.B.3** The dot product of two vectors is the sum of the products of their corresponding components. That is, $\langle a_1,b_1\rangle\cdot\langle a_2,b_2\rangle=a_1a_2+b_1b_2$.

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
4.8.C Determine a unit vector for a given vector.	▪ **4.8.C.1** A *unit vector* is a vector of magnitude 1. A unit vector in the same direction as a given nonzero vector can be found by scalar multiplying the vector by the reciprocal of its magnitude. ▪ **4.8.C.2** The vector $\langle a, b\rangle$ can be expressed as $a\vec{i}+b\vec{j}$ in R^2, where $\vec{i}$ and $\vec{j}$ are unit vectors in the x and y directions, respectively. That is, $\vec{i}=\langle 1,0\rangle$ and $\vec{j}=\langle 0,1\rangle$.
4.8.D Determine angles between vectors and magnitudes of vectors involved in vector addition.	▪ **4.8.D.1** The dot product is geometrically equivalent to the product of the magnitudes of the two vectors and the cosine of the angle between them. Therefore, if the dot product of two nonzero vectors is zero, then the vectors are perpendicular. ▪ **4.8.D.2** The Law of Sines and Law of Cosines can be used to determine side lengths and angles of triangles formed by vector addition.

TOPIC 4.9

Vector-Valued Functions

Instructional Periods: 1
Skills Focus: 3.C

LEARNING OBJECTIVES

4.9.A Represent planar motion in terms of vector-valued functions.

ESSENTIAL KNOWLEDGE

- **4.9.A.1** The position of a particle moving in a plane that is given by the parametric function $f(t)=(x(t), y(t))$ may be expressed as a *vector-valued function*, $p(t)=x(t)\vec{i}+y(t)\vec{j}$ or $p(t)=\langle x(t), y(t)\rangle$. The magnitude of the position vector at time t gives the distance of the particle from the origin.
- **4.9.A.2** The vector-valued function $v(t)=\langle x(t), y(t)\rangle$ can be used to express the velocity of a particle moving in a plane at different times, t. At time t, the sign of $x(t)$ indicates if the particle is moving right or left, and the sign of $y(t)$ indicates if the particle is moving up or down. The magnitude of the velocity vector at time t gives the speed of the particle.

TOPIC 4.10

Matrices

Instructional Periods: 2
Skills Focus: 1.B, 3.B

LEARNING OBJECTIVES

4.10.A Determine the product of two matrices.

ESSENTIAL KNOWLEDGE

- **4.10.A.1** An $n \times m$ matrix is an array consisting of n rows and m columns.
- **4.10.A.2** Two matrices can be multiplied if the number of columns in the first matrix equals the number of rows in the second matrix. The product of the matrices is a new matrix in which the component in the ith row and jth column is the dot product of the ith row of the first matrix and the jth column of the second matrix.

TOPIC 4.11

The Inverse and Determinant of a Matrix

Instructional Periods: 2
Skills Focus: 1.B, 3.B

LEARNING OBJECTIVES

4.11.A Determine the inverse of a 2×2 matrix.

ESSENTIAL KNOWLEDGE

- **4.11.A.1** The identity matrix, I, is a square matrix consisting of ones on the diagonal from the top left to bottom right and zeros everywhere else.
- **4.11.A.2** Multiplying a square matrix by its corresponding identity matrix results in the original square matrix.
- **4.11.A.3** The product of a square matrix and its inverse, when it exists, is the identity matrix of the same size.
- **4.11.A.4** The inverse of a 2×2 matrix, when it exists, can be calculated with or without technology.

LEARNING OBJECTIVES

4.11.B Apply the value of the determinant to invertibility and vectors.

ESSENTIAL KNOWLEDGE

- **4.11.B.1** The determinant of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$. The determinant can be calculated with or without technology and is denoted $\det(A)$.
- **4.11.B.2** If a 2×2 matrix consists of two column or row vectors from R^2, then the nonzero absolute value of the determinant of the matrix is the area of the parallelogram spanned by the vectors represented in the columns or rows of the matrix. If the determinant equals zero, then the vectors are parallel.
- **4.11.B.3** The square matrix A has an inverse if and only if $\det(A) \neq 0$.

TOPIC 4.12

Linear Transformations and Matrices

Instructional Periods: 1
Skills Focus: 1.B

LEARNING OBJECTIVES

4.12.A Determine the output vectors of a linear transformation using a 2×2 matrix.

ESSENTIAL KNOWLEDGE

- **4.12.A.1** A *linear transformation* is a function that maps an input vector to an output vector such that each component of the output vector is the sum of constant multiples of the input vector components.
- **4.12.A.2** A linear transformation will map the zero vector to the zero vector.
- **4.12.A.3** A single vector in R^2 can be expressed as a 2×1 matrix. A set of n vectors in R^2 can be expressed as a $2\times n$ matrix.
- **4.12.A.4** For a linear transformation, L, from R^2 to R^2, there is a unique 2×2 matrix, A, such that $L(\vec{v}) = A\vec{v}$ for vectors in R^2. Conversely, for a given 2×2 matrix, A, the function $L(\vec{v}) = A\vec{v}$ is a linear transformation from R^2 to R^2.
- **4.12.A.5** Multiplication of a 2×2 transformation matrix, A, and a $2\times n$ matrix of n input vectors gives a $2\times n$ matrix of the n output vectors for the linear transformation $L(\vec{v}) = A\vec{v}$.

TOPIC 4.13

Matrices as Functions

Instructional Periods: 3
Skills Focus: 1.B, 2.A, 3.A

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
4.13.A Determine the association between a linear transformation and a matrix.	▪ **4.13.A.1** The linear transformation mapping $\langle x, y\rangle$ to $\langle a_{11}x + a_{12}y, a_{21}x + a_{22}y\rangle$ is associated with the matrix $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$. ▪ **4.13.A.2** The mapping of the unit vectors in a linear transformation provides valuable information for determining the associated matrix. ▪ **4.13.A.3** The matrix associated with a linear transformation of vectors that maps every vector to the vector that is an angle θ counterclockwise rotation about the origin from the original vector is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. ▪ **4.13.A.4** The absolute value of the determinant of a 2×2 transformation matrix gives the magnitude of the dilation of regions in R^2 under the transformation.
4.13.B Determine the composition of two linear transformations.	▪ **4.13.B.1** The composition of two linear transformations is a linear transformation. ▪ **4.13.B.2** The matrix associated with the composition of two linear transformations is the product of the matrices associated with each linear transformation.
4.13.C Determine the inverse of a linear transformation.	▪ **4.13.C.1** Two linear transformations are inverses if their composition maps any vector to itself. ▪ **4.13.C.2** If a linear transformation, L, is given by $L(\vec{v}) = A\vec{v}$, then its inverse transformation is given by $L^{-1}(\vec{v}) = A^{-1}\vec{v}$, where A^{-1} is the inverse of the matrix A.

TOPIC 4.14

Matrices Modeling Contexts

Instructional Periods: 3
Skills Focus: 1.C, 3.B, 3.C

LEARNING OBJECTIVES	ESSENTIAL KNOWLEDGE
4.14.A Construct a model of a scenario involving transitions between two states using matrices.	▪ **4.14.A.1** A contextual scenario can indicate the rate of transitions between states as percent changes. A matrix can be constructed based on these rates to model how states change over discrete intervals.
4.14.B Apply matrix models to predict future and past states for n transition steps.	▪ **4.14.B.1** The product of a matrix that models transitions between states and a corresponding state vector can predict future states.
	▪ **4.14.B.2** Repeated multiplication of a matrix that models the transitions between states and corresponding resultant state vectors can predict the steady state, a distribution between states that does not change from one step to the next.
	▪ **4.14.B.3** The product of the inverse of a matrix that models transitions between states and a corresponding state vector can predict past states.

COURSE CHANGE PROPOSAL

Completed forms must be returned to the chief academic officer by ***October 1*** *to be considered for board approval.*

Date Initiated: 8/15/22 Administrator's Name: Jason Creel/Joshua Leinberger

Department and School: Information Technology—LakeView Technology Academy

Course Name: Mobile Game Programming

Request: ☒ New Course ☐ New Course Name ☐ Course Revision ☐ Remove Course

Credits: 3 *Check if honors:* ☐

Recommended Prerequisites (if any): None

Rationale: Explain why this course is needed. (If this is a course removal or name change, only fill out this section.)

This is one of three new courses being proposed for the 2023-24 school year. The new courses will expand the opportunity for information technology students to graduate with two Gateway certificates in programming. The Full Stack Web Developer certificate will prepare students for web software development from the front to back end of a web site. The Game Programming certificate gives students the skills to program two- and three-dimensional games using the programming techniques learned throughout their time at LakeView.

Proposed Course Description: In three or four sentences, write a course overview.

This course teaches students essential mobile game programming skills and develops skills to enhance their success in Gateway Technical College and their career. Students will gain a foundation for writing games on mobile devices. The class will utilize Android Studios to implement different parts of a game, such as graphics, user input, audio, animated objects, and physics. Putting these together students will work towards a finished product and working mobile game by the end of the course.

Content Standards and Benchmarks: List the primary content standards and benchmarks students will be expected to understand and be able to apply as a result of taking this course. (Attach additional documents as needed.)

1. Incorporate a software development kit into the development process.
2. Write code that draws to the screen.
3. Incorporate a graphics programming application programming interface into an application.
4. Produce 2-D transformations.
5. Program life cycle events.
6. Develop user input processes.
7. Integrate audio into a game.
8. Create a game development framework.
9. Code animated objects.
10. Utilize development languages.
11. Create collision detection method.

Scope and Sequence: Outline the planned structure for the course, including a tentative timeline for instruction. (Attach additional documents as needed.)

See attachment.

Cost Associated with the Course: Estimate the costs involved in offering this course. List desired texts and materials on a separate sheet. Also list and explain other needs.

A. Teaching Staff: $572.28 per student (Teaching and Learning budget)

B. Textbooks/Kits: $93 per student (Teaching and Learning budget) Textbook is a one-time purchase and shared with other courses.

C. Supplementary: $0

D. Facilities/Space: $0

E. Professional Learning: $0

SCHEDULE FOR MOBILE GAME PROGRAMMING	
Week 1	Unit 1—Orientation and Syllabus Unit 2—Android Applications Android Interface Review Shapes Assignment
Week 2	Unit 3—Chapter 7 (Images/Background) Images Assignment Unit 4—Game Loops and Input Animation In-Class Lab Touch Events In-Class Lab Animation Boundaries Assignment
Week 3	Unit 5—Game Engines Animation with Collision In-Class Lab Project Game Design Assignment Unit 6—Sprites Sprite Engine Demonstration Game Design Assignment
Week 4	Unit 7—Input Whack-A-Mole In-Class Lab **MIDTERM**
Week 5	Unit 8—Audio Whack-A-Mole Audio Addition Unit 9—Animation Animation In-Class Lab
Week 6	Unit 10—Matrix Transformations Game Engine SA Activity Sounds Activity Game Design Assignment
Week 7	Unit 11—Collision Game Engine with Collisions In-Class Lab Collisions Assignment
Week 8	Unit 12—Entity Grouping Entity Grouping In-Class Lab Entity Grouping Assignment Final Project
Week 9	**FINAL PROJECT** **FINAL EXAM**

APPENDIX C

COURSE CHANGE PROPOSAL

*Completed forms must be returned to the chief academic officer by **October 1** to be considered for board approval.*

Date Initiated: 8/15/22 Administrator's Name: Jason Creel/Joshua Leinberger

Department and School: Information Technology—LakeView Technology Academy

Course Name: Game Programming Technologies

Request: ☒ New Course ☐ New Course Name ☐ Course Revision ☐ Remove Course

Credits: 3 *Check if honors:* ☐

Recommended Prerequisites (if any): None

Rationale: Explain why this course is needed. (If this is a course removal or name change, only fill out this section.)

This is one of three new courses being proposed for the 2023-24 school year. The new courses will expand the opportunity for information technology students to graduate with two Gateway certificates in programming. The Full Stack Web Developer certificate will prepare students for web software development from the front to back end of a web site. The Game Programming certificate gives students the skills to program two- and three-dimensional games using the programming techniques learned throughout their time at LakeView.

Proposed Course Description: In three or four sentences, write a course overview.

Students will learn essential mobile game programming skills and develop skills to enhance their success in Gateway Technical College and their career. Students will gain a foundation for writing games on mobile devices. The class will utilize Android Studios to implement different parts of a game, such as graphics, user input, audio, animated objects, and physics. Putting these together students will work towards a finished product and working mobile game by the end of the course.

Content Standards and Benchmarks: List the primary content standards and benchmarks students will be expected to understand and be able to apply as a result of taking this course. (Attach additional documents as needed.)

Course competencies:

1. Incorporate a software development kit into the development process.
2. Write code that draws to the screen.
3. Incorporate a graphics programming application programming interface into an application.
4. Produce 2-D transformations.
5. Program life cycle events.
6. Develop user input processes.
7. Integrate audio into a game.
8. Create a game development framework.
9. Code animated objects.
10. Utilize development languages.
11. Create collision detection methods.

Scope and Sequence: Outline the planned structure for the course, including a tentative timeline for instruction. (Attach additional documents as needed.)

See attachment.

Cost Associated with the Course: Estimate the costs involved in offering this course. List desired texts and materials on a separate sheet. Also list and explain other needs.

A. Teaching Staff: $572.28 per student (Teaching and Learning budget)

B. Textbooks/Kits: $93 per student (Teaching and Learning budget) Textbook is a one-time purchase and shared with other courses.

C. Supplementary: $0

D. Facilities/Space: $0

E. Professional Learning: $0

SCHEDULE FOR GAME PROGRAMMING TECHNOLOGIES	
Week 1	Orientation and Syllabus Chapters 1 and 2 Types of Games Installing and Introduction Orientation to the Interface
Week 2	Chapters 3 and 4 Programming 2-D Games Programming 3-D Games Making a 2-D Platformer
Week 3	Chapters 5, 6, and 7 Static Mesh Actors Lighting Materials and Audio
Week 4	Chapters 8, 9, and 10 Landscapes and Worlds Space Invaders Gaming Levels Particles and Mesh Actors
Week 5	Chapters 11 and 12 Matinee, Cinematics, and Physics Placing Physics Based Objects Game Design **MIDTERM**
Week 6	Chapters 17 and 18 Blueprints, Variables, and Inputs Chapter Examples
Week 7	Review—Building a Complete Game Complete Arcade Shooter Demonstration
Week 8	Database Usage and Administration Controls and Queries File Sorting Database Demonstration Begin Building Final Project
Week 9	**FINAL PROJECT** **FINAL EXAM**

APPENDIX D

COURSE CHANGE PROPOSAL

*Completed forms must be returned to the chief academic officer by **October 1** to be considered for board approval.*

Date Initiated: 8/15/22 Administrator's Name: Jason Creel/Joshua Leinberger

Department and School: Information Technology—LakeView Technology Academy

Course Name: Front-End Development with Angular

Request: ☒ New Course ☐ New Course Name ☐ Course Revision ☐ Remove Course

Credits: 3 *Check if honors:* ☐

Recommended Prerequisites (if any): JavaScript

Rationale: Explain why this course is needed. (If this is a course removal or name change, only fill out this section.)

This is one of three new courses being proposed for the 2023-24 school year. The new courses will expand the opportunity for information technology students to graduate with two Gateway certificates in programming. The Full Stack Web Developer certificate will prepare students for web software development from the front to back end of a web site. The Game Programming certificate gives students the skills to program two- and three-dimensional games using the programming techniques learned throughout their time at LakeView.

Proposed Course Description: In three or four sentences, write a course overview.

This course introduces students to the Angular framework. Students will use TypeScript for developing Angular applications. Students learn the architecture of Angular applications including components, directives, services, and dependency injection framework. Students will create single-page applications (SPAs) that utilize data binding, the Angular router, and services and dependency injection, that are based on template-driven and reactive forms. Upon completion students will be able to create modern SPAs utilizing the Angular framework to solve a specific development need.

Content Standards and Benchmarks: List the primary content standards and benchmarks students will be expected to understand and be able to apply as a result of taking this course. (Attach additional documents as needed.)

1. Explore web application frameworks.
2. Investigate application design patterns.
3. Investigate the Angular architecture.
4. Create SPAs using Angular.
5. Create web applications using Bootstrap.
6. Create Angular components.
7. Code Angular applications using TypeScript.
8. Implement the Angular templating system.
9. Implement data binding in an Angular application.
10. Implement the Angular Router service.
11. Implement Angular Directives

Scope and Sequence: Outline the planned structure for the course, including a tentative timeline for instruction.

See attachment.

Cost Associated with the Course: Estimate the costs involved in offering this course. List desired texts and materials on a separate sheet. Also list and explain other needs.

A. Teaching Staff: $572.28 per student (Teaching and Learning budget)

B. Textbooks/Kits: $93 per student (Teaching and Learning budget) Textbook is a one-time purchase and shared with other courses.

C. Supplementary: $0

D. Facilities/Space: $0

E. Professional Learning: $0

SCHEDULE FOR FRONT-END DEVELOPMENT WITH ANGULAR	
Week 1	Orientation and Syllabus Before You Begin Chapter 1 Introduction to Angular PDF—TypeScript
Week 2	Chapter 2.1—Components Chapter 2.5—Modules Bootstrap Theme Assignment Nav and Footer Components Assignment
Week 3	Chapter 2.3—Directives Chapter 2.6—Understanding Data Binding JSON Data Assignment Chapter 9.2 Component Lifecycle PDF—Template Syntax Navigation Assignment
Week 4	Chapter 3—Router Basics Chapter 3—Practice Exercises Routing Assignment
Week 5	Chapter 8—Component Communications Component Communications Assignment **MIDTERM**
Week 6	Chapter 10—Template-driven and Reactive Forms Chapter 10—Book Exercises Template-Driven Form Assignment
Week 7	Chapter 11—Form Validations Chapter 11—Book Exercises Reactive Form Assignment Chapter 2.2—Services
Week 8	Chapter 5—Dependency Injection (DI) Chapter 5—Book Exercises DI Services Assignment Chapter 12—Interacting and Communicating with Servers DI CRUD Assignment **FINAL PROJECT**
Week 9	**FINAL PROJECT** **FINAL EXAM**

Kenosha Unified School District

Kenosha, Wisconsin

November 15, 2022

Preliminary 2023-24 Budget Discussion

With the adoption of the fiscal year 2022-23 budget now complete (approved October 25, 2022), the administration has begun the process of budget planning for the next fiscal which would start on July 1, 2023.

It is early in the process and many variables will be unknown for a while as we wait for the next State biennial budget to pass. However, the trending information that we do know at this time suggests the potential for a significant budgetary deficit for the upcoming 2023-24 fiscal year.

In the spirit of full transparency, tonight the administration will share and demonstrate some of the underlying data and assumptions that are causing concern:

- Projected 450 Full-Time Equivalent (FTE) enrollment loss.
- Hopeful scenario of $200 allowable per member change for revenue limit purposes.
    - This is a hopeful assumption of an inflationary increase in public education funding that is subject to debate within the State budget discussions.
    - If the enrollment projection holds true and we receive a $200 per member increase, we stand to lose an estimated $6 MM of revenue limit authority
- Health insurance premium increases at the contractually capped 11%.
    - Based on utilization, our experience is estimated to call for an increase as high as 30%, however, our contract contains an 11% cap.
    - The relevant (non-charter, non-grant) operational cost increase is estimated to be $3.6 MM
- The ACT 10 maximum allowed Consumer Price Index (CPI) increase to base wages is projected to be around 8% for collective bargaining agreements effective July 1, 2023.

This report is presented as an informational item only at this time to begin discussions.

Dr. Jeffrey Weiss	Tarik Hamdan
Superintendent of Schools	Chief Financial Officer

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Kenosha Unified School District

November 15, 2022

DONATIONS TO THE DISTRICT

The district has received the following donations:

1. The Forest County Potawatomi Community donated $50,000 to purchase new signage and/or paint for the Educational Support Center.

2. Lakeview Construction donated $1,000 to LakeView Technology Academy's Supermilage Vehicle Club.

Administrative Recommendation

Administration requests the Board of Education approve acceptance of the above-listed gift(s), grant(s), or bequest(s) as per Board Policy 1400 to authorize the establishment of appropriate accounts to monitor fiscal activity, to amend the budget to reflect this action and to publish the budget change per Wisconsin Statute 65.90(5)(a).

Dr. Jeffrey Weiss
Superintendent of Schools

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KENOSHA UNIFIED SCHOOL DISTRICT
Kenosha, Wisconsin

November 15, 2022

Tentative Schedule of Reports, Events, and Legal Deadlines for School Board November-December

November

- November 15, 2022 - Regular School Board Meeting – 7 P.M.
- November 23, 2022 – Schools Closed
- November 24-25, 2022 – Thanksgiving Recess – District Closed

December

- December 13, 2022 – Regular School Board Meeting – 7 P.M.
- December 16, 2022 – Staff Professional Learning, No Students Report
- December 23, 2022-January 2, 2023 – Winter Recess – District Closed

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