

## AP Calculus BC

**To:** All students enrolled in AP Calculus BC for the coming school year

**From:** AP Calculus BC teacher Mr. Topel

Going into AP Calculus BC, there are certain skills that have been taught to you over the previous years that are assumed that you have mastered. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

It is assumed that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example, should now be a part of you. If not, you would not be going into AP Calculus BC. The topics covered in the packet are skills that are used continually in AP Calculus BC.

The attached summer packet is for all students enrolled in AP Calculus BC in the fall. This packet will be collected on the first day of school and you will be tested on its contents. Show all work and check your solutions so that you can focus on refreshing the skills that you are lacking. Use the listed websites if and when you are unsure of how to work out any problems. Don't fake your way through these problems. As stated, students are notoriously weak in them, even students who have achieved well prior to AP Calculus BC. Use the websites.

This packet is intended to get you off to a good start in AP Calculus BC, and as stated earlier, you will be tested on this material at the start of the course in the fall. Spend some quality time with this packet this summer. Do not rely on a calculator. More than half of the AP exam is done without a calculator. It is a mistake to complete this immediately after school lets out for the summer. Let it go until midsummer. The idea is that these techniques are fresh in your mind at the start of the course in the fall. Likewise, it is a mistake to wait until the last minute to complete this as well. These problems do take time.

If you have questions about any of these problems or techniques used in solving them, you may contact Mr. Topel at [btopel@kUSD.edu](mailto:btopel@kUSD.edu).

*Have a great summer!*

Below are listed topics in the review. You can certainly do Google searches for any of these topics. Several sites are listed below that will cover almost all of these topics.

Here is a good site for most algebra topics:

<http://www.purplemath.com/modules/index.htm>

### **Beginning algebra topics**

Exponents

Negative and fractional exponents

### **Intermediate algebra topics**

Domain

Solving inequalities: absolute value

Solving inequalities: quadratic

Special factoring formulas

Function transformation

Factor theorem (p over q method)

Even and odd functions

Solving quadratic equations and the quadratic formula

### **Advanced algebra topics**

Asymptotes

Complex fractions

Composition of functions

Solving rational (fractional) equations

Here is a good site for trigonometric topics:

<http://www.mathematicshelpcentral.com/index.html> Once in the site, go to lecture notes.

### **Trigonometry topics**

Basic right triangle trigonometry

Unit circle

Trigonometric equations

Here is a good site for limits topics:

<http://www.calculus-help.com/funstuff/phobe.html>

### **Limits topics**

Evaluating limits

Techniques to evaluating limits

## AP CALCULUS BC SUMMER ASSIGNMENT

Have this assignment completed and the material that it covers understood by the first day of school. You will be tested on its contents when AP calculus starts.

1. Are the following statements true? If not, explain in words why not.

a)  $\frac{2k}{2x+h} = \frac{k}{x+h}$

b)  $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$

c)  $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

d)  $3\left(\frac{a}{b}\right) = \frac{3a}{3b}$

e)  $3\left(\frac{a}{b}\right) = \frac{3a}{b}$

f)  $3\left(\frac{a+b}{c}\right) = \frac{3a+b}{c}$

2. Simplify.

a)  $\frac{x/2}{x/4}$

b)  $h \div \frac{x+h}{h}$

c)  $\frac{\sqrt{x-2} + \frac{5}{\sqrt{x-2}}}{x-2}$

3. Solve for  $y'$ .

a)  $xy' + y = 1 + y'$

b)  $3y^2y' + 2yy' = 5y' + 2x$

c)  $3x^2yy' + 2xy^2 = 2yy'$

4. Solve the quadratic equation.

a)  $4x^2 - 21x - 18 = 0$

b)  $2x^2 - 3x + 3 = 0$

c)  $x^4 - 9x^2 + 8 = 0$

5. Write as a single fraction with the denominator in factored form.

a)  $\frac{7x^2 + 5x}{x^2 + 1} - \frac{5x}{x^2 - 6}$

b)  $20\left(\frac{2}{x+1} - \frac{3}{x}\right)$

6. Graph the equation  $y = x^3 - x$  and answer the following questions.

a) Is the point (3, 2) on the graph?

b) Is the point (2, 6) on the graph?

c) Is the function even, odd, or neither?

d) Find the x-intercepts.

7. Determine algebraically if the function is even, odd, or neither.

a)  $f(x) = 2x^2 - 7$

b)  $f(x) = -4x^3 - 2x$

c)  $f(x) = 4x^2 - 4x + 4$

8. Find the equation of the line that passes through the point (2, 4) and is parallel to the line  $2x + 3y - 8 = 0$ .

9. Find the equation of the line that is perpendicular to the line  $2x + 3y - 8 = 0$  at the point (1, 2).

10. The line with slope 5 that passes through the point  $(-1, 3)$  intersects the  $x$ -axis at a point. What are the coordinates of this point?

11. What are the coordinates of the point at which the line passing through the points  $(1, -3)$  and  $(-2, 4)$  intersects the  $y$ -axis?

12. Find  $f(1) - f(5)$  given  $f(x) = |x - 3| - 5$ .

13. Find  $f(x+2) - f(2)$  given  $f(x) = x^2 - 3x + 4$ .

14. Use interval notation to identify the domain for each of the following functions.

a)  $h(x) = \frac{1}{4x^2 - 21x - 18}$       b)  $k(x) = \sqrt{x^2 - 5x - 14}$       c)  $p(x) = \frac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$

d)  $d(x) = \ln(2x - 12)$

15. Find  $f(x+h)$  for  $f(x) = x^2 - 2x - 3$ .

16. Find  $\frac{f(x+h) - f(x)}{h}$  if  $f(x) = 8x^2 + 1$ .

17. Find  $\frac{f(x+h) - f(x)}{h}$  if  $f(x) = \frac{1}{x}$ .

18. Graph the function.

a)  $f(x) = \begin{cases} 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$       b)  $f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x+3 & (2, \infty) \end{cases}$       c)  $f(x) = \sqrt{16 - x^2}$

19. Given  $f(x) = x - 3$  and  $g(x) = \sqrt{x}$  complete the following.

a)  $f(g(x)) =$       b)  $g(f(x)) =$       c)  $f(f(x)) =$

20. Given  $f(x) = \frac{1}{x-5}$  and  $g(x) = x^2 - 5$  complete the following.

a)  $f(g(7)) =$       b)  $g(f(v)) =$       c)  $g(g(x)) =$

21. Let  $f(x) = 2x - 2$ . Complete the following.

- a) Graph  $f$ .
- b) Determine whether  $f$  has an inverse function.
- c) Graph  $f^{-1}$ .
- d) Give the equation for  $f^{-1}$ .

22. Simplify using only positive exponents. Do not rationalize the denominator.

a)  $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$       b)  $\left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{\frac{1}{2}}$       c)  $\left(\frac{x^{-2}}{y^{-1}} - x\right)^{-3}$

23. If  $f(x) = x^2 - 1$ , describe in words what the following would do to the graph of  $f(x)$ .

- a)  $f(x) - 4$       b)  $f(x - 4)$       c)  $-f(x + 2)$
- d)  $5f(x) + 3$       e)  $f(2x)$       f)  $|f(x)|$

24. Find the surface area of a box of height  $h$  whose base dimensions are  $p$  and  $q$ , and satisfies the following condition.

- a) The box is closed.
- b) The box has an open top.
- c) The box has an open top and a square base with side length  $p$ .

25. A seven foot ladder, leaning against a wall, touches the wall  $x$  feet above the ground. Write an expression in terms of  $x$  for the distance from the foot of the ladder to the base of the wall.

26. A piece of wire 5 inches long is to be cut into two pieces. One piece is  $x$  inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of  $x$ .

27. Evaluate. Be sure to answer in radians.

- a)  $\cos 0$       b)  $\sin 0$       c)  $\tan \frac{\pi}{2}$
- d)  $\cos \frac{\pi}{4}$       e)  $\sin \frac{\pi}{2}$       f)  $\sin \pi$
- g)  $\arcsin \frac{\sqrt{3}}{2}$       h)  $\arctan(-1)$       i)  $\arccos\left(-\frac{1}{2}\right)$

28. Find the solution of the equation for  $\theta \in [0, 2\pi)$ .

a)  $2\sin^2 \theta = 1 - \sin \theta$

b)  $2 \tan \theta - \sec^2 \theta = 0$

c)  $\sin 2\theta + \sin \theta = 0$

29. Which of the following expressions are identical?

a)  $\cos^2 x$

b)  $(\cos x)^2$

c)  $\cos x^2$

30. Which of the following expressions are identical?

a)  $(\sin x)^{-1}$

b)  $\arcsin x$

c)  $\sin x^{-1}$

d)  $\frac{1}{\sin x}$

31. Solve for  $x$ .

a)  $\ln e^3 = x$

b)  $\ln e^x = 4$

c)  $\ln x + \ln x = 0$

d)  $e^{\ln 5} = x$

e)  $\ln 1 - \ln e = x$

f)  $\ln 6 + \ln x - \ln 2 = 3$

g)  $\ln(x+5) = \ln(x-1) - \ln(x+1)$

32. Evaluate the limit.

a)  $\lim_{x \rightarrow 3} (x^2 + 2)$

b)  $\lim_{x \rightarrow 3} \frac{(x+3)(x-4)}{(x+3)(x+1)}$

c)  $\lim_{x \rightarrow 25} \frac{\sqrt{x}-5}{x-25}$

d)  $\lim_{x \rightarrow -2} \frac{x-4}{x^2-2x-8}$

e)  $\lim_{x \rightarrow -3} \frac{x^2+2x-3}{x^2+7x+12}$

f)  $\lim_{x \rightarrow -2} \frac{x^3+8}{x+2}$

g)  $\lim_{x \rightarrow 5} \frac{x-5}{|x-5|}$

h)  $\lim_{x \rightarrow 8} \frac{1}{x-8}$

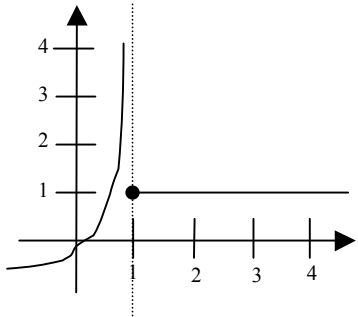
33. For each of the following determine i.)  $\lim_{x \rightarrow 1^-} f(x)$  ii.)  $\lim_{x \rightarrow 1^+} f(x)$  iii.)  $\lim_{x \rightarrow 1} f(x)$

a)  $f(x) = \begin{cases} x^2 - 1 & x < 1 \\ 4 - x & x \geq 1 \end{cases}$

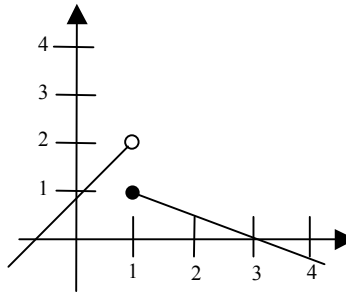
b)  $f(x) = \begin{cases} 3x - 1 & x \leq 1 \\ 3 - x & x > 1 \end{cases}$

c)  $f(x) = \begin{cases} -x^2 & x < 1 \\ 2 & x = 1 \\ x - 2 & x > 1 \end{cases}$

d)



e)



f)

